Strategies for problem solving

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Introduction to problem solving

In mathematics classes and in our everyday lives, we are presented with many problems and situations that can be solved using mathematical processes. When we 'solve problems', there are five main processes that we can use:

1. questioning
2. applying strategies
3. communicating
4. reasoning
5. reflecting.

To carry out a simple investigation, we may need to apply only two or three of the processes. For example, imagine you are asked to write up the results of an experiment. An electronic data logger has been measuring and storing air temperatures at a model house every hour, 24 hours a day, for 7 days. You might draw up a table with two columns — one for temperature and one for time. You'll need to decide whether to use seven tables, one for each day, or continue in one long column of temperatures. An explanation would be needed to communicate your strategy and findings. In this way, you would have focused on three processes:

1. questioning which strategy is most suitable
2. applying a strategy – you chose to use grid paper to summarise the data
3. communicating – you wrote a brief explanation to accompany the table so that your efforts could be checked and understood.

However, what if the results needed to be converted into graphical displays at a later date? What if you need to interpolate a temperature at a certain time of day? The information stored on the data logger could be downloaded onto a computer spreadsheet. The power of a spreadsheet would then be available to produce a huge variety of graphical displays, and would assist in the process of interpolating a certain temperature. If you had asked yourself these questions before applying a strategy, the result might have been a more detailed and useful record.

The five processes in problem solving are interrelated. By practising the skills involved in using all five processes, you will learn to tackle new mathematics problems with confidence and arrive at the correct and complete solution using the most appropriate methods.

Questioning

Why do you think that teachers often say: 'Read through the question a number of times before you begin'? What does this achieve? The aim is to make you think about the information that is needed and what will be the best approach to the problem. You are questioning and looking ahead to what may be produced by certain strategies. Be clear about the problem and the solution that is required. Consider whether you have come across a similar problem before that could be useful. Is there more than one method that you could use? Which method would be best to solve this problem?

Applying strategies

To apply a strategy when problem solving is to follow a plan or a method. The goal is to complete an investigation, answer a question or find a solution to a problem. Examples of strategies that can be used to achieve this include:

1. drawing up tables and diagrams
2. finding patterns of numbers
3. setting up equations and finding a solution
4. making use of technology, such as a computer spreadsheet
5. working backwards from the answer
6. using a process of elimination
7. looking at similar but simpler problems
8. using trial and error (guess and check).

Communicating

Another person who reads your work should be able to follow your method or strategy. It is important to learn to present data, explanations and solutions in a clear and concise form, using correct mathematical terms and appropriate diagrams.

Reasoning

When you reach a solution to a particular problem, make sure that you check it and can prove with mathematical reasoning that it is correct and that the method was appropriate. You may need to include reasoning with your explanations and solutions to justify your results.

Reflecting

Whenever you use the processes of problem solving, there is an opportunity to reflect on what you did and why, and to improve your understanding of the strategies and concepts. Consider whether the solution could have been obtained in a different or better way. Learn from the experience and look forward with confidence to using the knowledge gained when you are faced with the next problem or situation.

Throughout this textbook there are many questions and problems that can be answered using the five processes. On the following pages, some examples and practice questions are supplied. These will help you to focus on improving your skills relating to each process. The examples will also help you to become aware of the need to use all the processes to arrive at the correct solution in the most appropriate and efficient way.

Strategies for investigation and problem solving

Questioning skills

Before beginning work on a problem, devise some questions that ensure you set off on an effective path to solving it. Questions can help us to organise the information, test an idea, add another key piece of information and/or decide what to do next. There may be more than one question that could be asked.

The first important question is, 'What is this problem asking me to do or find?'. In order to answer this question, it is good practice to read the question at least twice. It may also be helpful to underline important facts or list the information given. This will help you identify these questions and lead you on the correct path to the required solution.

Create a table, then look for a pattern or a result

A table is a way of organising or grouping numbers. You should consider the number of rows and columns that will be needed and label them appropriately. A well-designed table helps you to see any patterns or results in the numbers you have organised, and also demonstrates to others how you were able to arrive at your solution. There are many different ways of presenting information in a table.

WORKED Example 1

Ahmed is investigating the suitability of the mathematical model $V = 2r^3$ when calculating the volume of a hemisphere. Ahmed wishes to determine for what values of r the model $V = 2r^3$ is within 2.5 cm^3 of the 'actual' volume for the hemisphere. Construct a table to investigate this problem.

THINK	WRITE
1 Read the question at least twice and take note of all the important facts.	Test the suitability of the formula $V = 2r^3$ as a possible model for the volume of a hemisphere (that is, half of a sphere). Compare the results obtained using the above formula with the actual formula: $V = \frac{2}{3}\pi r^3$ *Note:* $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ therefore $V_{\text{hemisphere}} = \frac{1}{2} \times \frac{4}{3}\pi r^3$ $= \frac{2}{3}\pi r^3$
2 Identify the solution required.	The solution involves determining which values of r produce differences that lie within 2.5 cm^3 of the actual volume.

THINK

3 Rule a table of values with column headings titled r, V_{model}, V_{actual} and Difference.

4 Assign r-values, such as 0.5, 1.0, 1.5, 2.0, 2.5, … to the first column. Allow space to add further rows if needed.

5 Generate values for each row of the table. For values of V_{model}, substitute each value of r into the formula $2 \times r^3$; for values of V_{actual}, substitute each value of r into the formula for volume of a hemisphere, that is, $\frac{2}{3} \times \pi \times r^3$.

6 Calculate the difference between the values in each row of the second and third columns to complete the fourth column.

7 Continue the table, looking for difference values that are no greater than 2.5 in magnitude.

8 As 2.8 may not be the best r-value, include $r = 2.9$ at the bottom of the table to check. Calculate the corresponding values across the row.

9 State the answer.

WRITE

r (cm)	V_{model} (cm^3)	V_{actual} (cm^3)	Difference (cm^3)
0.5	0.250	0.262	0.012
1.0	2.000	2.094	0.094
1.2	3.456	3.619	0.163
1.4	5.488	5.747	0.259
1.5	6.750	7.069	0.319
1.6	8.192	8.579	0.387
1.8	11.664	12.215	0.551
2.0	16.000	16.755	0.755
2.2	21.296	22.301	1.005
2.4	27.648	28.953	1.305
2.5	31.250	32.725	1.475
2.6	35.152	36.811	1.659
2.8	43.904	45.976	2.072
3.0	54.000	56.549	2.549
3.2	65.536	68.629	3.093
2.9	48.778	51.080	2.302

The volume of a hemisphere using the mathematical model $V = 2r^3$ is within 2.5 cm^3 of the actual volume of the hemisphere for radius values (to one decimal place) of 2.9 cm and less.

Try these

Construct a table to solve each of the following problems.

1 The actual formula for the circumference of a circle is $C = 2\pi r$. Hamish is testing the formula $C = 6r$. For what values of r (to 1 decimal place) does the real circumference differ from the tested value by less than 1 cm?

2 The formula for the surface area of a sphere is $SA = 4\pi r^2$. Test the formula $SA = 12r^2$. For what values of r (to one decimal place) is the difference calculated by these two formulas less than 2 cm^2?

3 Compare the y-values obtained for varying values of x using:
a $y = x^2 + 1$
b $y = 2^x$.
For what positive x-values is the difference between the corresponding y-values no greater than 2?

4 Compare the y-values obtained for varying values of x using:

a $y = \frac{1}{x}$ **b** $y = 2^{-x}$.

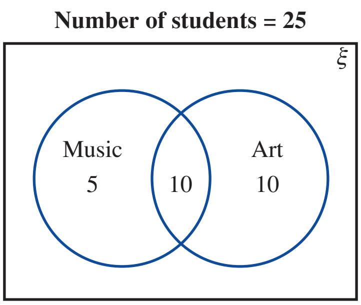
For what x-values is the difference between the corresponding y-values no greater than 2?

5 *Challenge:* Consider a cone that fits exactly into a *cube-shaped* box. Investigate the model $V = 2r^3$ and determine how well it determines the volume of this type of cone-shaped solid.

Draw a diagram, then look for a pattern or a result

When information is represented in the form of a diagram, it can be easier to study all the information at once. There are many types of diagrams, so no single diagram is necessarily the best.

If information can be separated into groups or sets and there is something in common between the sets, a Venn diagram can be used. For example, if we consider a class of 25 students where 15 students elect to study music and 20 elect to study art, we see that there must be some students who have elected to study both music and art. To create a Venn diagram for this situation, we would first draw a rectangle to represent all of the students in the class, and then we would draw two overlapping circles inside the rectangle to represent the students who have elected to study music and those who have elected to study art. The overlapping section would represent the students who have elected to study both. We can write the number of students in each section, but we need to ensure that the numbers written within any circle add to the required number.



WORKED Example 2

In Lam's class of 26 students, 13 play tennis and 20 play netball.

a How many students play both sports?

b Lam plays tennis only. How many other students play tennis only?

THINK	WRITE
a 1 Read the question at least twice and take note of all the important facts.	**a** In a class of 26 students, 13 play tennis and 20 play netball. Lam plays tennis only.
2 Identify the solution required.	The solution involves determining how many students play both sports and how many students besides Lam play tennis only.
3 To determine the number of students who play both sports, first add the number of students in each group. Adding 13 and 20 gives 33 which is 7 more than the 26 students in the class. This means that 7 students have been counted twice.	$13 + 20 = 33$ $33 - 26 = 7$ There are 7 students who have been counted twice.

THINK

4 Since there are students who play both tennis and netball, set up a Venn diagram with two overlapping circles. The rectangle represents the class of 26 students. The overlapping section represents the 7 students who play both sports. One circle needs to represent a total of 13 students who play tennis while the other needs to represent a total of 20 students who play netball. Fill in the required numbers.

5 Find the number of students who play both sports by reading the value from the Venn diagram.

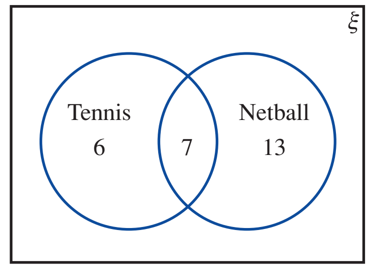
6 Answer the question.

b 1 Find the number of students who play tennis only by reading the value from the Venn diagram.

2 Answer the question.

WRITE

Number of students = 26



There are 7 students who play both tennis and netball.

b There are 6 students who play tennis only. Since Lam is one of these students, there are 5 other students who play tennis only.

Try these

Solve each of the following questions using a Venn diagram.

1 In a class of 22 students, each attends swimming practice, or lifesaving, or both. If 16 attend swimming practice and 9 attend lifesaving, find the number of students who attend both.

2 In a class of 30 students, each chooses to study French or German or both languages. If 24 study French and 18 study German, find the number of students who study both languages.

3 Vernon works in a restaurant where people choose to work Monday to Friday, or weekends, or Monday to Sunday. There are 15 employees altogether. If 13 work Monday to Friday and 14 work weekends, find the number of employees who *do not* work Monday to Sunday.

4 A jewellery store has 85 pieces of jewellery on display in its window. The range includes gold, silver and a mixture of gold and silver. Freshwater pearls are also featured in the display. There are 36 pieces containing gold, 45 pieces containing silver and 8 pieces with both silver and gold. How many pieces with freshwater pearls are in the display?

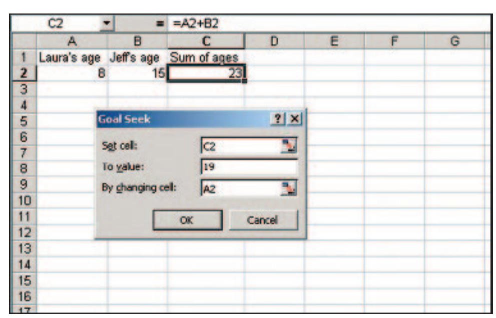
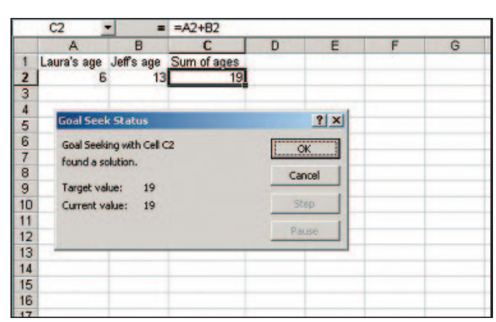
5 Most bread rolls in the school canteen are filled with ham, salad or ham and salad. Some have other fillings. Yesterday the canteen sold 284 bread rolls; 171 contained ham, while 165 contained salad. There were 44 rolls with other fillings sold. How many rolls with each type of filling were sold?

Set up equations and find a solution, making use of technology such as a computer spreadsheet

If there is an unknown quantity to be found, we can allocate a pronumeral to represent the unknown and form an equation. One way of solving the equation is to use a computer spreadsheet. We can enter the equation as a formula in a cell and type in headings in cells above to serve as labels.

WORKED Example 3

Jeff is 7 years older than Laura. Given that their ages add to 19, find Jeff's age.

THINK	WRITE
1 Read the question at least twice and take note of all the important facts.	Jeff is 7 years older than Laura. The sum of Jeff's age and Laura's age is 19 years.
2 Identify the solution required.	The solution involves determining Jeff's age.
3 Allocate pronumerals for Jeff's age and Laura's age.	Let J = Jeff's age, L = Laura's age
4 Write equations to connect the ages of Jeff and Laura.	$J = L + 7$ $J + L = 19$
5 Set up a spreadsheet and label the first cell of each column and enter the appropriate equations or formulas. Type 'Laura's age' in cell A1, and type any guess, say 8, into cell A2. Type 'Jeff's age' in cell B1, and type **=A2+7** in cell B2. This corresponds to the equation $J = L + 7$. Type 'Sum of ages' in cell C1, and type **=A2+B2** in cell C2. This corresponds to the equation $J + L = 19$.	
6 Highlight cell C2, click on **Tools** and **Goal Seek**. In the window that appears, enter 19 for 'To value:' and A2 for 'By changing cell:'.	
7 Click **OK** to produce the desired values.	
8 State the answer.	Jeff is 13 years old.

Try these

1 Benjamin is 4 years older than his sister. Twice Benjamin's age plus three times his sister's age equals 73. How old is Benjamin?

2 Elias and Shui are good friends. This year Elias is 48 years older than Shui. Nine years ago Elias was three times Shui's age. How old is Elias?

3 Gillian's coffee cups have exactly two-fifths the capacity of her tea cups. Using a 1.1 litre jug, Gillian filled five coffee cups and two tea cups exactly. What is the capacity of one coffee cup?

4 Consider two consecutive numbers. Twice the larger one plus three times the smaller one adds to 27. What are the numbers?

5 *Challenge:* The following question requires three equations: The shortest side of a triangle is 6.2 cm less than the longest side. The length of the remaining side is the average of the shortest and longest sides. The perimeter of the triangle is 26.7 cm. Find the length of the shortest side.

Work backwards from the answer

If there is a sequence of steps for which we know the final result, then a useful strategy may be to work backwards from this final result or answer. We start with the last step of the sequence.

WORKED Example 4

Carlos is a salesperson in a hardware store. He sold a kitchen sink that was on special at 25% off the recommended retail price because it was old stock. From the resulting price, the customer received a further 5% discount for paying cash. The customer paid $222.30. What was the recommended retail price of the kitchen sink?

THINK	WRITE
1 Read the question at least twice and take note of all the important facts.	The recommended retail price of a sink was reduced by 25%, followed by a further discount of 5%. The customer paid $222.30 for the sink.
2 Identify the solution required.	The solution involves determining the recommended retail price of the sink.
3 List the given information.	Let the recommended retail price be price A. So price A = ? Let the resulting price after the 25% discount be price B. Let the customer price after the 5% discount be price C. Price C = $222.30

Continued over page

THINK	WRITE
4 Use the price paid by the customer (price C) as the starting point. This amount is 95% of the resulting price after the 'old stock' discount of 25% (price B). Find price B.	95% of Price B = Price C $\frac{95}{100} \times \text{Price B} = 222.30$ $0.95 \times \text{Price B} = 222.30$ $\text{Price B} = \frac{222.30}{0.95}$ $= 234$
5 Use the results from steps 3 and 4 to calculate the recommended retail price (price A). *Note:* Price B is 75% of price A.	75% of Price A = Price B $\frac{75}{100} \times \text{Price A} = 234$ $0.75 \times \text{Price A} = 234$ $\text{Price A} = \frac{234}{0.75}$ $= 312$
6 Answer the question.	The recommended retail price was \$312.

Try these

1 Miatek sells imported electronic hi-fi equipment. As he cannot afford to keep too much stock, he offers a 40% discount on stock that is more than 12 months old. He offers a further 20% discount for multiple purchases (two or more items) of old stock. Amanda pays \$427.20 for a multiple purchase of hi-fi equipment that is 15 months old. What was the original price?

2 Luciano sells home-made pasta. From the original price he offers a discount of 15% to regular customers, then he adds 10% GST (Goods and Services Tax). Caterina is a regular customer. She pays \$5.61 per kg of pasta. What was the original price?

3 Rika works at a department store in the city. She adds a profit margin of 80% to the wholesale price to calculate the retail price. If stock is more than twelve months old she discounts the retail price by 25%. She then adds 10% GST. Solomon pays \$326.70 for a microfibre suit that is 18 months old. What was the wholesale price of the suit?

4 At the beginning of the year Joe received a pay rise of 5% of his previous year's salary. Because he had an excellent performance review in June, he received a bonus of 2% of his salary. His earnings for the year now amounted to \$66 980.34. What was Joe's salary before his pay rise at the beginning of the year?

5 Tony is a carpenter by trade. When he purchased a new hammer he decided on one that was already on special, being reduced by 15%. He received a further trade discount of 10%, as well as 5% discount for paying cash. The cash register showed the final price to be \$61.77. What was the original price of the hammer?

Use a process of elimination

When using a process of elimination we remove or eliminate possible solutions that do not match the given information. We first write all the possible combinations or solutions in a grid or table. From the information supplied, we cross out (eliminate) those combinations that do not match.

WORKED Example 5

Last Friday, the number of pizzas, p, made in a pizza shop was less than 100. There were d pizza delivery drivers. All drivers delivered some pizza but two drivers delivered 16 pizzas each. The variables p and d are connected by the equation $p(d - 3) = 102$. Find p, the number of pizzas made.

THINK	WRITE
1 Read the question at least twice and take note of all the important facts.	Less than 100 pizzas, p, were made on Friday. There were d delivery drivers. All drivers delivered some pizza. Two drivers delivered 16 pizzas each. The relationship between p and d is given by the equation $p(d - 3) = 102$.
2 Identify the solution required.	The solution involves finding p, the number of pizzas made.
3 Consider the values that p and d can take.	Both p and d must be positive whole numbers. Since p is positive then $(d - 3)$ must be positive to satisfy $p(d - 3) = 102$.
4 List some possible d values and calculate the matching p value obtained from the equation.	(see table below)

d	p
4	102
5	51
6	34
7	14.57
8	12.75
9	17
…	…

THINK	WRITE
5 Eliminate any values that do not match the given information.	Since p needs to be a whole number, $d \neq 7$ and $d \neq 8$. Since $p < 100$, $p \neq 102$ and so $d \neq 4$. Since two drivers delivered 16 pizzas each, $p > 32$. So $d \neq 6$ and $d \neq 9$ as there would be no pizzas left for the other drivers to deliver.
6 Check that the values that remain match the given information.	Hence $p = 51$ because $d = 5$. Both values match the given information.
7 Answer the question.	There were 51 pizzas made on Friday.

Try these

1 The number of pizzas, p, made in a pizza shop was less than 130. There were d pizza delivery drivers and all drivers delivered some pizza but three drivers delivered more than 23 pizzas each. The variables p and d are connected by the equation $p(d - 4) = 276$. Find p, the number of pizzas made.

2 The number of students, s, enrolled in a small country school is less than 106. There are c classes with at least 11 students per class. At all times, the total number of students in each class equals s. However, the variables s and c are connected by the equation $s(c + 4) = 132$. Find s, the number of students enrolled.

3 The number of sheep, s, shorn by a team of p professional shearers (that is, more than one shearer) was less than 563. The shearing shed accommodates no more than 12 shearers. All of the shearers managed to shear at least 40 sheep each. The variables s and p are connected by the equation $s(p - 7) = 1320$. Find s, the number of sheep that have been shorn.

4 Farmer Brown has a apricot trees and p peach trees in his orchard. He has fewer than 90 peach trees and more than 30 apricot trees. The relationship between a and p is represented by the equation $a(p + 6) = 2880$. How many of each type of tree does Farmer Brown have?

5 A car sales yard has n new cars and u used cars on display. There are never any more than 20 cars in the used car section. The new car area always displays at least 30 cars. The variables n and u are connected by the equation $n(u - 10) = 175$. How many new cars are displayed in the yard?

Look at similar but simpler problems

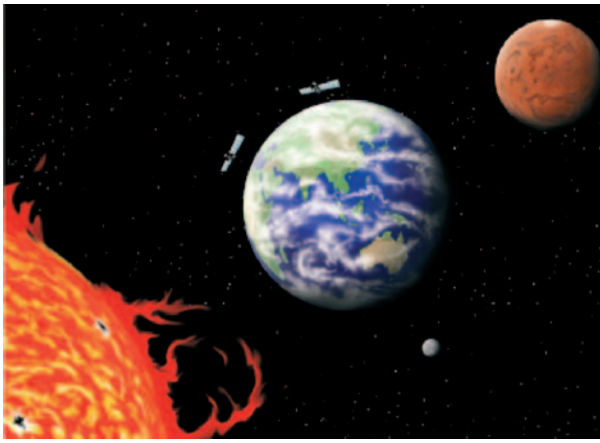
If you are overwhelmed by the size of the numbers involved in a question, try to solve a similar but simpler question. This can be achieved by changing the numbers in the original question to smaller numbers. After finding the answer to the simpler question, the same method can be used to solve the original problem.

WORKED Example 6

There are two satellites travelling in the same direction at different but constant speeds. Satellite A is travelling at a speed of 21 546 km/h. Another faster satellite, Satellite B, is 5840 km behind but it will overtake Satellite A in just 6 minutes. What is the speed of Satellite B?

THINK	WRITE
1 Read the question at least twice and take note of all the important facts.	Two satellites are travelling in the same direction at different but constant speeds. Satellite A travels at 21 546 km/h. Satellite B is 5840 km behind Satellite A. Satellite B overtakes Satellite A in 6 mins.
2 Identify the solution required.	The solution involves finding the speed of Satellite B.
3 Consider a simpler question with single-digit values.	First consider a simpler problem: Satellite A is travelling at a speed of 5 km/h. Another faster satellite, Satellite B, is 4 km behind but it will overtake Satellite A in 6 minutes. What is the speed of Satellite B?
4 Calculate how far Satellite A travels in 6 minutes.	Satellite A travels at 5 km/h. That is, Satellite A travels 5 km in 60 minutes or $(5 \div 10)$ km in 6 minutes, that is 0.5 km in 6 minutes.
5 Calculate how far Satellite B needs to travel in the same 6 minutes.	In the same 6 minutes, Satellite B needs to travel a total of 4 km and 0.5 km; that is, 4.5 km, to overtake Satellite A.

THINK	WRITE
6 Use the distance travelled in 6 minutes to calculate the speed at which Satellite B needs to travel to overtake Satellite A.	Speed of Satellite B = 4.5 km in 6 minutes = 45 km in 60 minutes = 45 km/h
7 Answer the question.	So Satellite B needs to travel at a speed of 45 km/h to overtake Satellite A.
8 Repeat steps 4 to 6 with the original information.	Using the original information: Satellite A travels at 21 546 km/h. That is, Satellite A travels 21 546 km in 60 minutes or (21 546 ÷ 10) km in 6 minutes, that is, 2154.6 km in 6 minutes. In the same 6 minutes, Satellite B needs to travel a total of 5840 km and 2 154.6 km; that is, 7994.6 km, to overtake Satellite A. Speed of Satellite B = 7994.6 km in 6 minutes = 79946 km in 60 minutes = 79946 km/h
9 Answer the question.	So Satellite B needs to travel at a speed of 79 946 km/h to overtake Satellite A.



Try these

1 There are two satellites travelling in the same direction at different but constant speeds. Satellite A is travelling at a speed of 21 546 km/h. A slower satellite, Satellite B, is 5840 km behind and it will be 9680 km behind Satellite A in just 12 minutes. What is the speed of Satellite B?

2 There are two satellites travelling in opposite directions at different but constant speeds. Satellite A is travelling at a speed of 21 546 km/h. A slower satellite, Satellite B, is 5840 km away and it will be 1680 km away from 1 Satellite A in just 12 minutes. What is the speed of Satellite B?

3 There are two spaceships travelling in opposite directions at different but constant speeds. Spaceship A is travelling at a speed of 21 546 km/h. A faster spaceship, Spaceship B, is 5840 km away and it will be 11 680 km away from Spaceship A in just 20 seconds. What is the speed of Spaceship B?

4 Two spaceships are 9850 km apart and travelling towards each other at constant speeds. The slower one is travelling at 10 800 km/h. After 10 seconds the two spaceships will be 9769 km apart. At what speed is the faster spaceship travelling?

5 Two spacecraft are seen to pass each other, travelling at constant speeds in opposite directions. After 5 minutes they are 2310 km apart. If the slower spacecraft is travelling at a speed of 12 600 km/h, what is the speed of the faster spacecraft?

Use trial and error (guess and check), making use of technology such as a computer spreadsheet

Sometimes it may not be easy to solve a problem directly; in this case we can use a strategy by which we guess at the solution. We test this value (guess), using the available information supplied in the problem, to check whether it is the solution. Even if it is not the solution to the problem, this process provides us with further information that we can use to try another, better-informed guess.

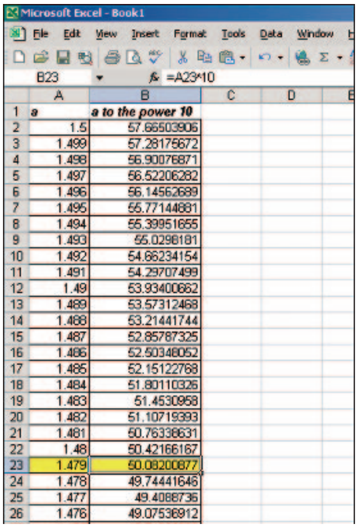
We can continue to guess and check until we reach the solution. Since this can be a lengthy process, we can use technology such as a spreadsheet to provide instant feedback on our checking.

WORKED Example 7

Pedro is working on a mathematical model. He needs to find a positive value of a that satisfies the equation $a^x = 50$ when x is exactly 10.

THINK / **WRITE**

1. Read the question at least twice and take note of all the important facts.
   The equation of the mathematical model is $a^x = 50$ and $x = 10$.
2. Identify the solution required.
   The solution involves using a spreadsheet to find a positive value of a which makes the equation $a^{10} = 50$ true.
3. Write the equation with the given value for x.
   $a^{10} = 50$
4. Substitute a few different values of a to see how close a^{10} is to 50. Try $a = 2$.
   $2^{10} = 1024$
5. Try $a = 1$.
   $1^{10} = 1$
6. Since it appears that the value of a is between 1 and 2, try $a = 1.5$.
   $1.5^{10} = 57.665 \ldots$
7. As the value of a needs to be a little less than 1.5, set up a spreadsheet to calculate a^{10} for a values very close to 1.5.



	A	B
1	a	a to the power 10
2	1.5	57.66503906
3	1.499	57.28175672
4	1.498	56.90076871
5	1.497	56.52206282
6	1.496	56.14562689
7	1.495	55.77144881
8	1.494	55.39951655
9	1.493	55.0298181
10	1.492	54.66234154
11	1.491	54.29707499
12	1.49	53.93400662
13	1.489	53.57312468
14	1.488	53.21441744
15	1.487	52.85787325
16	1.486	52.50348052
17	1.485	52.15122768
18	1.484	51.80110326
19	1.483	51.4530958
20	1.482	51.10719393
21	1.481	50.76338631
22	1.48	50.42166167
23	1.479	50.08200877
24	1.478	49.74441646
25	1.477	49.4088736
26	1.476	49.07536912

8. Write the answer.
   From the spreadsheet of values, $a = 1.479$ is a very good approximation for the solution to the equation $a^{10} = 50$.

Try these

1 Heathcliffe is working on a mathematical model. He needs to find a positive value of a that satisfies the equation $a^x = 0.5$ when x is exactly 9. Use a spreadsheet to find a.

2 Elice is working on a mathematical model. She needs to find a positive value of a that satisfies the equation $a^{-x} = 0.5$ when x is exactly 9. Use a spreadsheet to find a.

3 Hayden is working on a mathematical model. He needs to find a positive value of a that satisfies the equation $a^{\frac{1}{x}} = 0.5$ when x is exactly 9. Use a spreadsheet to find a.

4 Justine is working on a mathematical model. She needs to find a positive value of a that satisfies the equation $a^{\sqrt{x}} = 10$ when x is exactly 10. Use a spreadsheet to find a.

5 Todd is working on a mathematical model. He needs to find a positive value of a that satisfies the equation $a^{\frac{1}{\sqrt{x}}} = 5$ when x is exactly 5. Use a spreadsheet to find a.

Communicating, reasoning and reflecting

Once you have applied a strategy, there remains the task of communicating your findings. (You are usually expected to support your findings with some reasoning.) It is important to understand that solving problems involves much more than just writing numbers on a page. Words should accompany the mathematics and these words should be in the form of appropriate English and use correct mathematical terms. A danger area occurs with the use of the equals (=) sign. Care should be taken when this sign is used, that the mathematics on either side are indeed equal. It is important to communicate correctly in all cases — not just in solving problems. After providing a solution to a question, it is good practice to review your solution to see whether another person could understand your work without first reading the question.

Finally, if you take the time to reflect on your work, you may increase your understanding of the problem and the strategies you used to solve it. You may be able to connect this to previous experiences as well as to future problems you will have to tackle.

In summary, to solve problems successfully, the following steps should be followed.

1. Read the question at least twice and take note of all the important facts.
2. Identify the solution required.
3. Decide on, and apply an appropriate strategy (create a table, draw a diagram, use technology, work backwards, use a process of elimination, look at similar but simpler problems or use trial and error).
4. Communicate the whole solution using appropriate language and mathematical terms.
5. Support the solution with mathematical reasoning.
6. Reflect on the solution. Does it answer the question and does it make sense?

WORKED Example 8

The town hall clock is exactly 6 minutes fast. It is gaining 3 seconds every 2 hours. Generate a pattern from which you could determine how fast the clock is at any given time.

THINK

1. Read the question at least twice and take note of all the important facts.
2. Identify the solution required.
3. Consider using a strategy that will generate a pattern of clock times. A computer spreadsheet will do the job much faster and neater than paper and pencil would and allows us to continue the pattern for whatever length of time we wish. A graph may be needed later.
4. Set up a spreadsheet with columns for 'hours elapsed', 'extra seconds gained' and 'total minutes fast'. Since the clock gains 3 seconds every 2 hours, this means that the clock gains 1.5 seconds every hour. In cell A2, type 1 for the first hour elapsed; in cell B2, type the formula **=A2*1.5** and in cell C2, type the formula **=6+B2/60**. Highlight and **Fill Down** the columns.
5. Communicate the answer with reasoning.
6. Reflect on the answer.

WRITE

1. The clock is exactly 6 minutes fast and it gains 3 seconds every 2 hours.
2. The solution involves generating a pattern to show how fast the clock is at any given time.
3. A spreadsheet will be used so that the pattern can be extended as far as needed. A graph could also be produced if needed.
4. Spreadsheet:

	A	B	C
1	hours elapsed	extra seconds gained	total minutes fast
2	1	=1.5*A2	=6+B2/60
3	2	3	6.05
4	3	4.5	6.075
5	4	6	6.1
6	5	7.5	6.125
⋮		⋮	
41	40	60	7
42	41	61.5	7.025
43	42	63	7.05
44	43	64.5	7.075
45	44	66	7.1
46	45	67.5	7.125
⋮		⋮	
91	90	135	8.25
92	91	136.5	8.275
93	92	138	8.3
94	93	139.5	8.325
95	94	141	8.35
96	95	142.5	8.375

5. The time gained can be found using the spreadsheet above. For example, the clock has gained 8.25 minutes; that is, 8 minutes and 15 seconds after operating for 90 hours.
6. This clock cannot show the correct time, but it is always possible to calculate the actual time because the amount of time gained in seconds per hour is known. In this respect the clock is reliable.

Try these

1 The formula for the perimeter of a semicircle is $P = \pi r + 2r$. Lyn is testing the formula $P = 5r$. For what values of r (to 2 decimal places) does the actual perimeter differ from the tested value by less than 1 cm?

2 A survey to determine whether Year 9 students preferred the Media Studies or Graphic Design elective showed that 75 liked Media Studies, 59 liked Graphic Design and 12 didn't like either. Of the 123 students, what number liked both electives?

3 Sarah is 4 years older than Anna. The sum of their ages is 38 years. Use a computer spreadsheet to find Sarah's age.

4 The Handy Home Renovators Suppliers reduced the price of all their items by 10% at their annual winter sale. Gordon purchased a bathroom vanity unit and received a further 10% discount for paying cash. He paid \$656.10. What was the original price of the vanity unit?

5 The Botanical Gardens has p pine trees and e eucalypt trees in their grounds. The relationship between these two variables is represented by the equation $p(e - 7) = 120$. There are fewer than 19 eucalypt trees and more than 15 pine trees. How many of each type of tree is in the Botanical Gardens?

6 Two satellites are travelling towards each other at different but constant speeds. The satellites are 8925 km apart with the faster satellite travelling at a speed of 25 200 km/h. After a period of 5 seconds the satellites are seen to be 8860 km apart. What is the speed of the slower satellite?

7 Jennifer is working on a mathematical model. She needs to find a positive value of a that satisfies the equation $a^{x^2} = 2$ when x is exactly 2. Use a spreadsheet to find the value of a, correct to 3 decimal places.

8 A water tank holds 400 litres. Every night, 20 litres is added automatically even if no-one is using any water. The water tank is losing 1 mL every second due to a dripping tap but this is the only water used. Will the tank run out of water? If so, how many days will this take?

9 In a 50-metre pool, three swimmers start a 20-lap race together. Swimmer A is the fastest, taking 30 seconds for each 50 metres; swimmer B takes 30.5 seconds and swimmer C takes 32.5 seconds. Will any of the swimmers touch either end of the pool simultaneously before the race ends?

10 Amelia and Aana are both preparing for the athletics javelin competition. Each day, Amelia improves her throw of the javelin by 1.9 cm. Aana improves by 2.4 cm each day for 4 days, but on the fifth day she worsens her throw by 1.5 cm. If the girls start out the same, with a throw of 17 metres, which girl is the first to throw 18 metres? How many more days does the second girl need to practise after the first girl achieves a throw of 18 metres?

11 Julie has employed a pastry cook to work in her café. Julie's costs are $7 per hour plus $1 per pastry. Julie's business has competition from another business, Albert's Bookshop, across the road. Albert's Bookshop also sells coffee and pastries. Albert pays his pastry chef $18 per hour, but for every pastry sold, Albert's costs reduce by $1. Consider Albert's costs and how they depend on the number of pastries sold. Consider Julie's costs and how they depend on the number of pasties sold.

a Is there a linear relationship between Julie's costs (y) and the number of pastries sold (x).

b Conduct a similar investigation for Albert's Bookshop.

c *Challenge:* Consider the possibility that on a particular day both cafés sell the same number of pastries. Investigate if a particular number of pastries sold by both cafés results in Albert's costs and Julie's costs being the same.

12 Angelique and Tony are deciding how many rooms of their house they can afford to have air-conditioned for $2000. The cost will be $500 plus $15 per cubic metre of space to be air-conditioned. Air-conditioning costs are dependent on the volume of the room(s). Each room in the house is 3.2 metres wide and 2.4 metres high but the length of the rooms vary. The lengths of the rooms being considered are as follows:
2.6 m, 2.8 m, 3 m, 3.5 m, 4 m and 4.2 m.

a Is there a linear relationship between air-conditioning costs (y) and the volume of the rooms?

b Is there a linear relationship between the total volume of the selected rooms and the sum of the lengths of the selected rooms?

c Is there a linear relationship between the air-conditioning costs and the sum of the lengths of the rooms selected to be air-conditioned (x)?

d How many rooms could be air-conditioned for a budget of $2000?

e If the family room with a length of 4.2 m is the most important room for air-conditioning, which additional rooms should be selected for air-conditioning in order that the most number of rooms are selected without exceeding the $2000 budget?