Quadratic equations

4

When dolphins are travelling very quickly, they use less energy if they leap out of the water. The length and height of a dolphin's leap has been modelled by the equation $h = -4d^2 + d$ where h is the dolphin's height above water and d is the horizontal distance from its starting point. Both h and d are in metres. What horizontal distance does the dolphin cover in one leap?

The equation above is a quadratic equation. In *Maths Quest 9* you learnt how to solve a quadratic equation by factorising the expression and applying the Null Factor Law. In this chapter you will learn further methods of solving quadratic equations and how to determine if a solution to a quadratic equation exists at all.

Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest 10* CD-ROM or ask your teacher for a copy.

SkillSHEET 4.1

Expanding brackets

1 Expand each of the following.

a $4(3x + 5)$ **b** $5x(2x - 3)$ **c** $-4x(3 - 2x)$

SkillSHEET 4.2

Expanding a pair of brackets

2 Expand each of the following expressions.

a $(x + 2)(x - 2)$ **b** $(2x - 3)^2$ **c** $(3x + 2)(2x - 5)$

SkillSHEET 4.3

Factorising by taking out the highest common factor

3 Factorise each of the following expressions.

a $4x^2 + 8x$ **b** $-15x^2 - 9x$ **c** $6x^2 - x$

SkillSHEET 4.4

Factorising by taking out a common binomial factor

4 Factorise each of the following.

a $3x(x + 2) + 4(x + 2)$ **b** $4x(x - 1) - (x - 1)$ **c** $-2x(x + 3) - (x + 3)$

SkillSHEET 4.5

Finding a factor pair that adds to a given number

5 Find a factor pair of the first number that adds to the second number.

a 6, 5 **b** –4, 0 **c** 3, –4

SkillSHEET 4.7

Multiplication of fractions

6 Perform the following multiplications.

a $\frac{1}{8} \times \frac{2}{3}$ **b** $1\frac{1}{2} \times 1\frac{1}{3}$ **c** $2\frac{1}{3} \times 1\frac{13}{14}$

SkillSHEET 4.8

Division of fractions

7 Calculate each of the following.

a $\frac{1}{4} \div \frac{1}{2}$ **b** $\frac{6}{7} \div \frac{2}{21}$ **c** $1\frac{3}{4} \div 2\frac{5}{8}$

SkillSHEET 4.9

Simplifying algebraic fractions

8 Write each of the following fractions in simplest form.

a $\dfrac{(x+3)(x-2)}{(x-2)}$ **b** $\dfrac{x+7}{(x-3)(x+7)^2}$ **c** $\dfrac{3x(x+2)^2(x+3)}{6x(x+3)^2(x+2)}$

SkillSHEET 4.11

Simplifying surds

9 Simplify each of the following.

a $\sqrt{24}$ **b** $3\sqrt{12}$ **c** $4\sqrt{243}$

Expanding algebraic expressions

To change an algebraic expression from a factorised form to an expanded form, we remove the brackets by multiplying terms. In chapter 2 we revised some of the simpler expansions involving a single set of brackets where we multiply each term inside the bracket by whatever is outside the bracket.

When there are two sets of brackets this is known as a binomial expansion. To expand a binomial, multiply each term in the first bracket by each term in the second.

WORKED Example 1

Expand each of the following.

a $(x + 2)(x - 5)$ **b** $(x - 7)(6 - x)$

THINK	WRITE
a 1 Write the expression.	**a** $(x + 2)(x - 5)$
2 Multiply the terms in the second bracket by the first term in the first bracket and then the second term in the first bracket.	$= x(x - 5) + 2(x - 5)$
3 Remove the brackets by multiplying each term in the brackets by the term outside the bracket.	$= x^2 - 5x + 2x - 10$
4 Collect like terms.	$= x^2 - 3x - 10$
b 1 Write the expression.	**b** $(x - 7)(6 - x)$
2 Multiply the terms in the second bracket by the first term in the first bracket and then the second term in the first bracket. Notice that the minus sign stays with the second term in the first bracket (–7).	$= x(6 - x) - 7(6 - x)$
3 Remove the brackets by multiplying each term in the brackets by the term outside the bracket. Remember to change the sign when the term outside the bracket is negative.	$= 6x - x^2 - 42 + 7x$
4 Collect like terms.	$= -x^2 + 13x - 42$

An alternative method

The word **FOIL** provides us with an acronym for the expansion of a binomial product.

Each letter is the first letter of a word that is a reminder of how to find the four original terms before collecting like terms.

First: multiply the **first** terms in each bracket together $\overset{F}{(x + a)(x - b)}$

Outer: multiply the two **outer** terms $\overset{O}{(x + a)(x - b)}$

Inner: multiply the two **inner** terms I $(x + a)(x - b)$

Last: multiply the **last** terms in each bracket together L $(x + a)(x - b)$

O F

$(x + a)(x - b)$

I L

WORKED Example 2

Use FOIL to expand $(x + 2)(x - 5)$.

THINK	WRITE
1 Write the expression.	$(x + 2)(x - 5)$
2 Multiply the first term in each bracket, then the outer terms, the inner terms and finally the last two terms.	$= x \times x + x \times -5 + 2 \times x + 2 \times -5$ $= x^2 - 5x + 2x - 10$
3 Collect like terms.	$= x^2 - 3x - 10$

If there is a term outside the pair of brackets, expand the brackets and then multiply each term of the expansion by that term.

WORKED Example 3

Expand $3(x + 8)(x + 2)$.

THINK	WRITE
1 Write the expression.	$3(x + 8)(x + 2)$
2 Use FOIL to expand the pair of brackets.	$= 3(x^2 + 2x + 8x + 16)$
3 Collect like terms within the brackets.	$= 3(x^2 + 10x + 16)$
4 Multiply each of the terms inside the brackets by the term outside the brackets.	$= 3x^2 + 30x + 48$

This method can be extended to include three or even more sets of brackets. In such examples we expand two brackets first, then we multiply the result by the third bracket.

Expanding expressions which are perfect squares

A special binomial expansion involves the expansion of a perfect square. In *Maths Quest 9* we used a special rule that allows quick expansion of a perfect square. Consider the following expansion.

$$\begin{aligned}&(a + b)^2\\&= (a + b)(a + b)\\&= a^2 + ab + ab + b^2\\&= a^2 + 2ab + b^2\end{aligned}$$

This result tells us that to expand a perfect square we:

1. square the first term
2. multiply the two terms together and double
3. square the last term.

Similarly $(a - b)^2 = a^2 - 2ab + b^2$. (Try this expansion for yourself.)

Any perfect square can be expanded using FOIL; however, this rule provides a quicker method of performing such expansions.

Expanding perfect squares
$$(a+b)^2 = a^2 + 2ab + b^2$$
OR
$$(a-b)^2 = a^2 - 2ab + b^2$$

WORKED Example 4

Expand and simplify each of the following.

a $(2x-5)^2$ **b** $-3(2x+7)^2$

THINK

a 1 Write the expression.
2 Expand using the rule $(a-b)^2 = a^2 - 2ab + b^2$.

b 1 Write the expression.
2 Expand using the rule $(a+b)^2 = a^2 + 2ab + b^2$.
3 Multiply every term inside the brackets by the term outside the brackets.

WRITE

a $(2x-5)^2$
$= (2x)^2 - 2 \times 2x \times 5 + (5)^2$
$= 4x^2 - 20x + 25$

b $-3(2x+7)^2$
$= -3[(2x)^2 + 2 \times 2x \times 7 + (7)^2]$
$= -3(4x^2 + 28x + 49)$
$= -12x^2 - 84x - 147$

Difference of two squares rule

A similar result can be found when we expand a product of the form $(a+b)(a-b)$ where the two brackets contain the same terms but one has the terms added and the other subtracted. (*Note:* The brackets can be in any order.) This produces what is called the difference of two squares rule.

$(a+b)(a-b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

Can you see why this rule has this name?

Difference of two squares rule $(a+b)(a-b) = a^2 - b^2$

WORKED Example 5

Expand and simplify each of the following.

a $(3x+1)(3x-1)$ **b** $4(2x-7)(2x+7)$

THINK

a 1 Write the expression.
2 Expand using the rule $(a+b)(a-b) = a^2 - b^2$.

b 1 Write the expression.
2 Expand using the difference of two squares rule.
3 Multiply every term inside the brackets by the term outside the brackets.

WRITE

a $(3x+1)(3x-1)$
$= (3x)^2 - (1)^2$
$= 9x^2 - 1$

b $4(2x-7)(2x+7)$
$= 4[(2x)^2 - (7)^2]$
$= 4(4x^2 - 49)$
$= 16x^2 - 196$

remember

1. When expanding an algebraic expression with:
   (a) one bracket — multiply each term inside the bracket by the term outside the bracket
   (b) two brackets — multiply the terms in order: First terms, Outer terms, Inner terms and then Last terms
   (c) a term outside the two brackets — expand the pair of brackets first, then multiply each term of the expanded expression by the term outside the brackets
   (d) three brackets — expand any two of the brackets and then multiply the expanded expression by the third bracket.
2. Perfect squares rule: $(a + b)^2 = a^2 + 2ab + b^2$ or $(a - b)^2 = a^2 - 2ab + b^2$
3. Difference of two squares rule: $(a + b)(a - b) = a^2 - b^2$

EXERCISE 4A Expanding algebraic expressions

SkillSHEET 4.1 Expanding brackets

EXCEL Spreadsheet Expanding $a(bx + c)$

1 Expand each of the following.

a $2(x + 3)$ **b** $4(x - 5)$ **c** $3(7 - x)$
d $-(x + 3)$ **e** $x(x + 2)$ **f** $2x(x - 4)$
g $3x(5x - 2)$ **h** $5x(2 - 3x)$ **i** $2x(4x + 1)$
j $2x^2(2x - 3)$ **k** $3x^2(2x - 1)$ **l** $5x^2(3x + 4)$

WORKED Example 1, 2

SkillSHEET 4.2 Expanding a pair of brackets

2 Expand each of the following.

a $(x + 3)(x - 4)$ **b** $(x + 1)(x - 3)$ **c** $(x - 7)(x + 2)$
d $(x - 1)(x - 5)$ **e** $(2 - x)(x + 3)$ **f** $(x - 4)(x - 2)$
g $(2x - 3)(x - 7)$ **h** $(x - 1)(3x + 2)$ **i** $(3x - 1)(2x - 5)$
j $(3 - 2x)(7 - x)$ **k** $(5 - 2x)(3 + 4x)$ **l** $(11 - 3x)(10 + 7x)$

WORKED Example 3

EXCEL Spreadsheet Expanding $(ax + b)(cx + d)$

3 Expand each of the following.

a $2(x + 1)(x - 3)$ **b** $4(2x + 1)(x - 4)$ **c** $-2(x + 1)(x - 7)$
d $2x(x - 1)(x + 1)$ **e** $3x(x - 5)(x + 5)$ **f** $6x(x - 3)(x + 3)$
g $-2x(3 - x)(x - 3)$ **h** $-5x(2 - x)(x - 4)$ **i** $6x(x + 5)(4 - x)$

Mathcad Expanding algebraic expressions

4 Expand each of the following.

a $(x - 1)(x + 1)(x + 2)$ **b** $(x - 3)(x - 1)(x + 2)$ **c** $(x - 5)(x + 1)(x - 1)$
d $(x - 1)(x - 2)(x - 3)$ **e** $(2x - 1)(x + 1)(x - 4)$ **f** $(3x + 1)(2x - 1)(x - 1)$

5 Expand each of the following and simplify.

a $(x + 2)(x - 1) - 2x$ **b** $3x - (2x - 5)(x + 2)$
c $(2x - 3)(x + 1) + (3x + 1)(x - 2)$ **d** $(3 - 2x)(2x - 1) + (4x - 5)(x + 4)$
e $(x + 1)(x - 7) - (x + 2)(x - 3)$ **f** $(x - 2)(x - 5) - (x - 1)(x - 4)$
g $(x - 3)(x + 1) + \sqrt{3}\,x$ **h** $(\sqrt{2} - 3x)(\sqrt{3} + 2x) - \sqrt{5}\,x$

6 **multiple choice**

a $(3x - 1)(2x + 4)$ expands to:

A $6x^2 + 10x - 4$ **B** $5x^2 - 24x + 3$ **C** $3x^2 + 2x - 4$
D $6x^2 - 10x - 4$ **E** $6x^2 - 4$

b $-2x(x - 1)(x + 3)$ expands to:

A $x^2 + 2x - 3$ **B** $-2x^2 - 4x + 6$ **C** $-2x^3 - 4x^2 + 6x$
D $-2x^3 + 4x^2 - 6x$ **E** $-2x^3 - 3$

WORKED Example 4a

7 Expand and simplify each of the following.

a $(x - 1)^2$ **b** $(x + 2)^2$ **c** $(x + 5)^2$
d $(4 + x)^2$ **e** $(7 - x)^2$ **f** $(12 - x)^2$
g $(3x - 1)^2$ **h** $(12x - 3)^2$ **i** $(5x + 2)^2$
j $(2 - 3x)^2$ **k** $(5 - 4x)^2$ **l** $(1 - 5x)^2$

EXCEL Spreadsheet Expanding $(ax + b)^2$

WORKED Example 4b

8 Expand and simplify each of the following.

a $2(x - 3)^2$ **b** $4(x - 7)^2$ **c** $3(x + 1)^2$
d $-(2x + 3)^2$ **e** $-(7x - 1)^2$ **f** $2(2x - 3)^2$
g $-3(2 - 9x)^2$ **h** $-5(3 - 11x)^2$ **i** $-4(2x + 1)^2$

WORKED Example 5

9 Expand and simplify each of the following.

a $(x + 7)(x - 7)$ **b** $(x + 9)(x - 9)$ **c** $(x - 5)(x + 5)$
d $(x - 1)(x + 1)$ **e** $(2x - 3)(2x + 3)$ **f** $(3x - 1)(3x + 1)$
g $(7 - x)(7 + x)$ **h** $(8 + x)(8 - x)$ **i** $(3 - 2x)(3 + 2x)$

EXCEL Spreadsheet Expanding $(ax + b)(cx + d)$

10 The length of the side of a rectangle is $(x + 1)$ cm and the width is $(x - 3)$ cm.

a Find an expression for the area of the rectangle.
b Simplify the expression by expanding.
c If $x = 5$ cm, find the dimensions of the rectangle and hence its area.

11 Chickens are kept in a square enclosure with sides measuring x m. If the number of chickens is increased, the size of the enclosure is to have 1 metre added to one side and 2 metres to the other side.

a Draw a diagram of the original enclosure.
b Add to the first diagram or draw another one to show the new enclosure. Mark the lengths of each side on your diagram.
c Find an expression for the area of the new enclosure.
d Simplify the expression by removing the brackets.
e If the original enclosure had sides of 2 metres, find the area of the original square and then the area of the bigger enclosure.

12 A jewellery box has a square base with sides measuring $(x + 2)$ cm and is 5 cm high.

a Write an expression for the area of the base of the box.
b Write an expression for the volume of the box. (V = area of base × height)
c Simplify the expression by expanding the brackets.
d If $x = 8$ cm, find the volume of the box in cm^3.
e Find the area of the lid of the box and, hence, find how many 1-cm square tiles could be inlaid in the lid.

GAME time Quadratic equations — 001

Factorising expressions with two or four terms

The most straightforward type of factorisation is where a common factor is removed from the expression. Once this has been done, we need to consider the number of terms in the expression to see the type of factorisation that may be possible.

Factorising expressions of the type $a^2 - b^2$

When factorising an algebraic expression of the type $a^2 - b^2$, follow these steps.

1. Look for a common factor first. If there is one, factorise by taking it out.
2. Rewrite the expression showing the two squares and identifying the a and b parts of the expression.
3. Factorise, using the rule $a^2 - b^2 = (a + b)(a - b)$

WORKED Example 6

Factorise each of the following.

a $4x^2 - 9$ **b** $7x^2 - 448$ **c** $x^2 - 17$

THINK	WRITE
a 1 Write the expression.	**a** $4x^2 - 9$
2 Check for a common factor and write as two perfect squares.	$= (2x)^2 - 3^2$
3 Factorise using $a^2 - b^2 = (a + b)(a - b)$.	$= (2x + 3)(2x - 3)$
b 1 Write the expression.	**b** $7x^2 - 448$
2 Check for a common factor and take it out.	$= 7(x^2 - 64)$
3 Write the terms in the bracket as two perfect squares.	$= 7(x^2 - 8^2)$
4 Factorise using $a^2 - b^2 = (a + b)(a - b)$.	$= 7(x + 8)(x - 8)$
c 1 Write the expression.	**c** $x^2 - 17$
2 Check for a common factor and write as two perfect squares. In this case a surd needs to be used to rewrite 17 as a perfect square.	$= x^2 - (\sqrt{17})^2$
3 Factorise using $a^2 - b^2 = (a + b)(a - b)$.	$= (x + \sqrt{17})(x - \sqrt{17})$

Note: This method requires a subtraction sign between the two perfect squares.

Factorising expressions with four terms

If there are four terms to be factorised, look for a common factor first. Then group the terms in pairs and look for a common factor in each pair. It may be that a new common factor emerges as a bracket (common binomial factor).

WORKED Example 7

Factorise each of the following.

a $x - 4y + mx - 4my$

b $x^2 + 3x - y^2 + 3y$

THINK	WRITE
a 1 Write the expression and look for a common factor. (There isn't one.)	**a** $x - 4y + mx - 4my$
2 Group the terms so that those with common factors are next to each other.	$= (x - 4y) + (mx - 4my)$
3 Take out a common factor from each group.	$= 1(x - 4y) + m(x - 4y)$
4 Factorise by taking out a common binomial factor. The factor $(x - 4y)$ is common to both groups.	$= (x - 4y)(1 + m)$
b 1 Write the expression and look for a common factor.	**b** $x^2 + 3x - y^2 + 3y$
2 Group the terms so that those that can be appropriately factorised are next to each other.	$= (x^2 - y^2) + (3x + 3y)$
3 Factorise each group.	$= (x + y)(x - y) + 3(x + y)$
4 Factorise by taking out a common binomial factor. The factor $(x + y)$ is common to both groups.	$= (x + y)(x - y + 3)$

In worked example 7, we looked at grouping terms in pairs. This is known as grouping 'two and two'. Now we will look at grouping a different combination, known as grouping 'three and one'.

WORKED Example 8

Factorise the following expression $x^2 + 12x + 36 - y^2$.

THINK	WRITE
1 Write the expression and look for a common factor.	$x^2 + 12x + 36 - y^2$
2 Group the terms so that those that can be factorised are next to each other.	$= (x^2 + 12x + 36) - y^2$
3 Factorise the quadratic trinomial.	$= (x + 6)(x + 6) - y^2$ $= (x + 6)^2 - y^2$
4 Factorise the expression using $a^2 - b^2 = (a + b)(a - b)$.	$= (x + 6 + y)(x + 6 - y)$

remember

1. To factorise an expression with two terms:
   (a) take out any common factors
   (b) check whether the difference of two squares rule can be used.
2. To factorise an expression with four terms:
   (a) take out any common factors
   (b) check whether they can be grouped using the 'two and two' method or the 'three and one' method.

EXERCISE 4B Factorising expressions with two or four terms

SkillSHEET 4.3 Factorising by taking out the highest common factor

EXCEL Spreadsheet Factorising $ax^2 + bx$

SkillSHEET 4.4 Factorising by taking out a common binomial factor

EXCEL Spreadsheet Difference of 2 squares

Mathcad Factorising two or four terms

1 Factorise each of the following by taking out a common factor.

a $x^2 + 3x$ b $x^2 - 4x$ c $3x^2 - 6x$
d $4x^2 + 16x$ e $9x^2 - 3x$ f $8x - 8x^2$
g $12x - 3x^2$ h $8x - 12x^2$ i $8x^2 - 11x$

2 Factorise each of the following by taking out a common binomial factor.

a $3x(x - 2) + 2(x - 2)$ b $5(x + 3) - 2x(x + 3)$
c $(x - 1)^2 + 6(x - 1)$ d $(x + 1)^2 - 2(x + 1)$
e $(x + 4)(x - 4) + 2(x + 4)$ f $7(x - 3) - (x + 3)(x - 3)$

WORKED Example 6a

3 Factorise each of the following.

a $x^2 - 1$ b $x^2 - 9$ c $x^2 - 25$
d $x^2 - 100$ e $y^2 - k^2$ f $4x^2 - 9y^2$
g $16a^2 - 49$ h $25p^2 - 36q^2$ i $1 - 100d^2$

WORKED Example 6b

4 Factorise each of the following.

a $4x^2 - 4$ b $5x^2 - 80$ c $ax^2 - 9a$
d $2B^2 - 8D^2$ e $100x^2 - 1600$ f $3ax^2 - 147a$
g $4px^2 - 256p$ h $36x^2 - 16$ i $108 - 3x^2$

5 **multiple choice**

a If the factorised expression is $(x + 7)(x - 7)$, then the original expression must have been:

A $x^2 - 7$ B $x^2 + 7$ C $x^2 - 49$ D $x^2 + 49$ E $x^2 - 14x + 49$

b If the factorised expression is $\left(\frac{x}{4} - \frac{3}{5}\right)\left(\frac{x}{4} + \frac{3}{5}\right)$ then the original expression must have been:

A $\frac{x^2}{4} - \frac{3}{5}$ B $\frac{x^2}{16} - \frac{9}{25}$ C $\frac{x^2}{4} - \frac{(\sqrt{3})^2}{(\sqrt{5})^2}$

D $\frac{x^2}{4} - \frac{9}{25}$ E $\frac{x^2}{16} - \frac{(\sqrt{3})^2}{(\sqrt{5})^2}$

c The factorised form of $64x^2 - 9y^2$ is:

A $(64x + 9y)(64x - 9y)$ B $(8x + 3y)(8x - 3y)$ C $(8x - 3y)(8x - 3y)$
D $(8x + 3y)(8x + 3y)$ E $(16x + 3y)(16x - 3y)$

WORKED Example 6c

6 Factorise each of the following.

a $x^2 - 11$ **b** $x^2 - 7$ **c** $x^2 - 15$
d $4x^2 - 13$ **e** $9x^2 - 19$ **f** $3x^2 - 66$
g $5x^2 - 15$ **h** $2x^2 - 4$ **i** $12x^2 - 36$

7 Factorise each of the following expressions:

a $(x - 1)^2 - 4$ **b** $(x + 1)^2 - 25$ **c** $(x - 2)^2 - 9$
d $(x + 3)^2 - 16$ **e** $49 - (x + 1)^2$ **f** $36 - (x - 4)^2$
g $(x - 1)^2 - (x - 5)^2$ **h** $4(x + 2)^2 - 9(x - 1)^2$ **i** $25(x - 2)^2 - 16(x + 3)^2$

8 The area of a rectangle is $(x^2 - 25)$ cm^2.

a Factorise the expression.
b Using the factors, find a possible length and width of the rectangle.
c If $x = 7$ cm, find the dimensions of the rectangle.
d Hence, find the area of the rectangle.
e If $x = 13$ cm, how much bigger would the area of this rectangle be?

9 A circular garden of diameter $2r$ m is to have a gravel path laid around it. The path is to be 1 m wide.

a Find the radius of the garden.
b Find the radius of the circle that includes both garden and path.
c Find the area of the garden in terms of r.
d Find the area of the garden and path together in terms of r, using the formula for the area of a circle.
e Write an equation to find the area of the path, then write your equation in fully factorised form.
f If the radius of the garden is 5 m, use the answer to **e** to find the area of the path, correct to 2 decimal places.

WORKED Example 7a

10 Factorise each of the following.

a $x - 2y + ax - 2ay$ **b** $2x + ax + 2y + ay$ **c** $ax - ay + bx - by$
d $4x + 4y + xz + yz$ **e** $ef - 2e + 3f - 6$ **f** $mn - 7m + n - 7$
g $6rt - 3st + 6ru - 3su$ **h** $7mn - 21n + 35m - 105$ **i** $64 - 8j + 16k - 2jk$
j $3a^2 - a^2b + 3ac - abc$ **k** $5x^2 + 10x + x^2y + 2xy$ **l** $2m^2 - m^2n + 2mn - mn^2$

11 Factorise each of the following.

a $xy + 7x - 2y - 14$ **b** $mn + 2n - 3m - 6$ **c** $pq + 5p - 3q - 15$
d $s^2 + 3s - 4st - 12t$ **e** $a^2b - cd - bc + a^2d$ **f** $xy - z - 5z^2 + 5xyz$

WORKED Example 7b

12 Factorise each of the following.

a $a^2 - b^2 + 4a - 4b$ **b** $p^2 - q^2 - 3p + 3q$ **c** $m^2 - n^2 + lm + ln$
d $7x + 7y + x^2 - y^2$ **e** $5p - 10pq + 1 - 4q^2$ **f** $49g^2 - 36h^2 - 28g - 24h$

WORKED Example 8

13 Factorise each of the following.

a $x^2 + 14x + 49 - y^2$ **b** $x^2 + 20x + 100 - y^2$ **c** $a^2 - 22a + 121 - b^2$
d $9a^2 + 12a + 4 - b^2$ **e** $25p^2 - 40p + 16 - 9t^2$ **f** $36t^2 - 12t + 1 - 5v$

14 multiple choice

a In the expression $3(x - 2) + 4y(x - 2)$, the common binomial factor is:

A $3 + 4y$ **B** $3 - 4y$ **C** x **D** $-x + 2$ **E** $x - 2$

b Which of the following terms is a perfect square?

A 9 **B** $(x + 1)(x - 1)$ **C** $3x^2$ **D** $5(a + b)^2$ **E** $25x$

c Which of the following expressions can be factorised using grouping?

A $x^2 - y^2$ **B** $1 + 4y - 2xy + 4x^2$ **C** $3a^2 + 8a + 4$

D $x^2 + x + y - y^2$ **E** $2a + 4b - 6ab + 18$

Factorising expressions with three terms

An expression with three terms is called a **trinomial**. **Quadratic trinomials** can be written in the form $ax^2 + bx + c$ where the highest power is a squared term.

Factorising $ax^2 + bx + c$ when $a = 1$

The following method works for every possible trinomial (with $a = 1$) that can be factorised.

Step 1 Place the trinomial in the correct order or standard form $x^2 + bx + c$.

Step 2 Find all the factor pairs of c (the constant term).

Step 3 Identify the factor pair whose sum equals b.

Step 4 Express the trinomial $x^2 + bx + c$ in factor form, that is $(x + \_\_)(x + \_\_)$.

Remember to always check first for any common factors. You can check your answer by expanding the brackets.

WORKED Example 9

Factorise each of the following.

a $x^2 - x - 20$ **b** $-2x^2 + 16x - 14$

THINK	WRITE
a 1 Write the expression.	**a** $x^2 - x - 20$
2 Check for a common factor (none).	
3 Identify the factors of x^2 as x and x and find the factors of the last term (−20) which add to equal the coefficient of the middle term (−1).	-20: $5 + -4 = 1$, $-5 + 4 = -1$
4 Write the expression and its factorised form.	$x^2 - x - 20 = (x - 5)(x + 4)$
b 1 Write the expression.	**b** $-2x^2 + 16x - 14$
2 Check for a common factor. (−2 can be taken out.)	$= -2(x^2 - 8x + 7)$
3 Identify the factors of x^2 as x and x and find the factors of the last term (7) which add to equal the coefficient of the middle term (−8).	7: $1 + 7 = 8$, $-1 + -7 = -8$
4 Write the expression and its factorised form.	$-2(x^2 - 8x + 7) = -2(x - 1)(x - 7)$

Factorising $ax^2 + bx + c$ when $a \neq 1$

If the **coefficient** of x^2 is not 1, and there is not a common factor, we factorise the expression by splitting up the x-term so that the expression can then be factorised by grouping.

A quadratic trinomial of the form $ax^2 + bx + c$ is broken up into four terms by finding two numbers which multiply to give ac and add to b.

Alternatively the cross-product method could be used.

WORKED Example 10

Factorise $10x^2 - x - 2$ by **a** grouping **b** the cross-product method.

THINK / **WRITE**

a

1. Write the expression and check for a common factor (none).

   a $10x^2 - x - 2$

2. Find the factor pair of ac (-20) which gives a sum of b (-1).

   -20: $2 + -10 = -8$, $-4 + 5 = 1$, $4 + -5 = -1$

3. Rewrite the expression by breaking the x-term into two terms using the factor pair from step 2.

   $10x^2 - x - 2$
   $= 10x^2 + 4x - 5x - 2$

4. Factorise by grouping terms.

   $= 2x(5x + 2) - (5x + 2)$
   $= (5x + 2)(2x - 1)$

b

1. Write the expression.

   b $10x^2 - x - 2$

2. List the factor pairs of the first term $(10x^2)$.
   Note: There are two possible factor pairs: x and $10x$ or $2x$ and $5x$. In some cases, both will have to be tested in order to obtain the required middle term.

   Factors of

   $\mathbf{10x^2}$ $\mathbf{-2}$

   $2x$ -2
   $5x$ 1 $2x - 10x = -8x$

3. List the factor pairs of the last term (-2).

   $2x$ -1
   $5x$ 2 $\mathbf{4x - 5x = -x}$

4. Calculate the sum of each cross product pair.

   $2x$ 1
   $5x$ -2 $-4x + 5x = x$

5. Select the combination that produces the correct middle term.

   $2x$ 2
   $5x$ -1 $-2x + 10x = 8x$

6. Express the trinomial in factor form.
   Note: The first bracket contains the first row entries and the second bracket the second row entries that produce the correct middle term.

   $10x^2 - x - 2 = (2x - 1)(5x + 2)$

remember

1. Whenever factorising any expression, look for a common factor first.
2. To factorise a quadratic trinomial when the coefficient of x^2 is 1 (that is, $x^2 + bx + c$):
   (a) identify the factor pair of c whose sum is equal to b
   (b) express the trinomial in factor form, $x^2 + bx + c = (x + \_\_)(x + \_\_)$.
3. To factorise a quadratic trinomial when the coefficient of x^2 is not 1 (that is, $ax^2 + bx + c$ where $a \neq 1$):
   (a) identify the factor pair of ac that has a sum of b
   (b) rewrite the expression by breaking the x-term into two terms using the factor pair from the previous step
   (c) factorise the resulting expression by grouping.
   Alternatively, the cross-product method could be used to solve any quadratic trinomial.
4. All factorisations can be checked by expanding.

EXERCISE 4C Factorising expressions with three terms

SkillSHEET 4.5 Finding a factor pair that adds to a given number

WORKED Example 9a

1 Factorise each of the following.

a $x^2 + 3x + 2$ **b** $x^2 + 4x + 3$ **c** $x^2 + 10x + 16$
d $x^2 + 8x + 16$ **e** $x^2 - 2x - 3$ **f** $x^2 - 3x - 4$
g $x^2 - 11x - 12$ **h** $x^2 - 4x - 12$ **i** $x^2 + 3x - 4$
j $x^2 + 4x - 5$ **k** $x^2 + 6x - 7$ **l** $x^2 + 3x - 10$
m $x^2 - 4x + 3$ **n** $x^2 - 9x + 20$ **o** $x^2 + 9x - 70$

EXCEL Spreadsheet Factorising $x^2 + bx + c$

WORKED Example 9b

2 Factorise each of the following.

a $-2x^2 - 20x - 18$ **b** $-3x^2 - 9x - 6$ **c** $-x^2 - 3x - 2$
d $-x^2 - 11x - 10$ **e** $-x^2 - 7x - 10$ **f** $-x^2 - 13x - 12$
g $-x^2 - 7x - 12$ **h** $-x^2 - 8x - 12$ **i** $2x^2 + 14x + 20$
j $3x^2 + 33x + 30$ **k** $5x^2 + 105x + 100$ **l** $5x^2 + 45x + 100$

Mathcad Factorising three terms

3 Factorise each of the following.

a $a^2 - 6a - 7$ **b** $t^2 - 6t + 8$ **c** $b^2 + 5b + 4$
d $m^2 + 2m - 15$ **e** $p^2 - 13p - 48$ **f** $c^2 + 13c - 48$
g $k^2 + 22k + 57$ **h** $s^2 - 16s - 57$ **i** $g^2 - g - 72$
j $v^2 - 28v + 75$ **k** $x^2 + 14x - 32$ **l** $x^2 - 19x + 60$

4 **multiple choice**

a To factorise $-14x^2 - 49x + 21$, the first step is to:
A find factors of 14 and 21 that will add to make -49
B take out 14 as a common factor
C take out -7 as a common factor
D find factors of 14 and -49 that will add to make 21
E take out -14 as a common factor

b The expression $42x^2 - 9x - 6$ can be completely factorised as:
A $(6x - 3)(7x + 2)$ **B** $3(2x - 1)(7x + 2)$ **C** $(2x - 1)(21x + 6)$
D $3(2x + 1)(7x - 2)$ **E** $42(x - 3)(x + 2)$

WORKED Example 10

5 Factorise each of the following using an appropriate method.

a $2x^2 + 5x + 2$ **b** $2x^2 - 3x + 1$ **c** $4x^2 - 17x - 15$
d $4x^2 + 4x - 3$ **e** $2x^2 - 9x - 35$ **f** $3x^2 + 10x + 3$
g $6x^2 - 17x + 7$ **h** $12x^2 - 13x - 14$ **i** $10x^2 - 9x - 9$
j $20x^2 + 3x - 2$ **k** $12x^2 + 5x - 2$ **l** $15x^2 + x - 2$

6 Factorise each of the following, remembering to look for a common factor first.

a $4x^2 + 2x - 6$ **b** $9x^2 - 60x - 21$ **c** $72x^2 + 12x - 12$
d $-18x^2 + 3x + 3$ **e** $-60x^2 + 150x + 90$ **f** $24ax^2 + 18ax - 105a$
g $-8x^2 + 22x - 12$ **h** $-10x^2 + 31x + 14$ **i** $-24x^2 + 35x - 4$
j $-12x^2 - 2xy + 2y^2$ **k** $-30x^2 + 85xy + 70y^2$ **l** $-600x^2 - 780xy - 252y^2$

7 Consider the expression $(x - 1)^2 + 5(x - 1) - 6$.

a Substitute $w = x - 1$ in this expression.
b Factorise the resulting quadratic.
c Replace w with $x - 1$ and simplify each factor. This is the factorised form of the original expression.

8 Use the method outlined in question **7** to factorise each of the following expressions.

a $(x + 1)^2 + 3(x + 1) - 4$ **b** $(x + 2)^2 + (x + 2) - 6$
c $(x - 3)^2 + 4(x - 3) + 4$ **d** $(x + 3)^2 + 8(x + 3) + 12$
e $(x - 7)^2 - 7(x - 7) - 8$ **f** $(x - 5)^2 - 3(x - 5) - 10$

9 Students decide to make Valentine's Day cards as a fundraising activity. They make the cards so that the area is equal to $x^2 - 4x - 5$.

a Factorise the expression to find the dimensions of the cards in terms of x.
b Write down the length of the shorter side in terms of x.
c If the shorter sides of a card are 10 cm and the longer sides are 16 cm, find the value of x.
d Find the area of this particular card.
e If they want to make 3000 Valentine's Day cards, how much cardboard will they require? Give the answer in terms of x.

WorkSHEET 4.1

THINKING

Quilt squares

A quilt is made by repeating the block at right. The letters indicate the colours of fabric that make up the block — yellow, black and white. The yellow and white pieces are square, and the black pieces are rectangular. The blocks are sewn together in rows and columns. The finished quilt, made from 100 blocks, is a square with an area of 1.44 m^2.

y	b	y
b	w	b
y	b	y

An interesting feature is created when the blocks are sewn together: each colour forms a shape. The shape and its area are exactly the same for each colour (The feature is created throughout the quilt, except at the edges.).

1 Determine the size of each yellow, black and white fabric piece in a block.

2 How much (in m^2) of each of the different colours would be needed to construct the quilt? (Ignore seam allowances.)

3 Draw a sketch to show the design of the finished product.

Sydney 2000 Olympic mascots

The letter beside each quadratic expression and its factorised form gives the puzzle code.

A $= x^2 - 4x - 5$
$=$

B $= x^2 + 5x + 6$
$=$

C $= x^2 + 7x + 12$
$=$

D $= x^2 - 7x + 10$
$=$

E $= x^2 - 9x + 18$
$=$

H $= 2x^2 + 2x - 4$
$=$

I $= 3x^2 + 21x + 30$
$=$

K $= 2x^2 - 10x - 72$
$=$

L $= 3x^2 - 27x + 42$
$=$

N $= 2x^2 - 5x - 3$
$=$

O $= 2x^2 + 7x - 4$
$=$

P $= 2x^2 - 5x - 12$
$=$

R $= 2x^2 + x - 6$
$=$

S $= 3x^2 + 11x - 4$
$=$

T $= 3x^2 - 8x - 3$
$=$

U $= 4x^2 + 4x - 15$
$=$

W $= 6x^2 - 5x - 6$
$=$

Y $= 6x^2 + 11x - 10$
$=$

$(2x-3)(x+2)$	$(x-3)(x-6)$	$(x+1)(x-5)$	$(x-2)(x-5)$	$(x+2)(x+3)$	$(x+1)(x-5)$	$(x+3)(x+4)$	$2(x-9)(x+4)$	$(3x+2)(2x-3)$	$(x+1)(x-5)$	$(2x-3)(x+2)$	$(x-2)(x-5)$	$(3x-1)(x+4)$

$(x+1)(x-5)$	$(2x-3)(x+2)$	$(2x-3)(x+2)$	$(2x-3)(2x+5)$	$(x+2)(x+3)$	$(x+1)(x-5)$	$2(x-9)(x+4)$	$(2x-1)(x+4)$	$(2x-1)(x+4)$	$2(x-9)(x+4)$

$(3x-1)(x+4)$	$(2x-3)(2x+5)$	$(2x+3)(x-4)$	$(2x+5)(3x-2)$	$(3x+1)(x-3)$	$(x+1)(x-5)$	$3(x-2)(x-7)$	$(2x+3)(x-4)$

$(x+1)(x-5)$	$(2x+1)(x-3)$	$(x-2)(x-5)$	$3(x+2)(x+5)$	$2(x-1)(x+2)$	$(x+3)(x+4)$	$(x-3)(x-6)$

MATHS QUEST CHALLENGE

1 **Consider the following pattern.**

$2^2 - 1^2 = 3 = 2 + 1$

$3^2 - 2^2 = 5 = 3 + 2$

$4^2 - 3^2 = 7 = 4 + 3$

$5^2 - 4^2 = 9 = 5 + 4$

a **Use this pattern to write the answer for $87^2 - 86^2$.**

b **Explain how this pattern works.**

2 **Explain why every prime number, with two exceptions, must be one more or one less than a multiple of 6. What are the two exceptions?**

Factorising by completing the square

Consider factorising $x^2 - 8x + 5$. Can you find factors of 5 that add to −8? There are no integer factors but there are factors.

So far we have factorised quadratic trinomials where the factors have involved integers. There are cases, however, where not only rational numbers are used, but also irrational numbers such as surds.

WORKED Example 11

Complete the square for $x^2 - 6x$.

THINK	WRITE
1 Write the expression.	$x^2 - 6x$
2 Identify the coefficient of x. Halve it and square the result.	$x^2 - 6x + \left(\frac{1}{2} \times -6\right)^2$
3 Add the result to the given terms to produce a new expression.	$x^2 - 6x + 9$
4 Factorise this new expression to produce a perfect square.	$= (x - 3)^2$

Completing the square for the first two terms of a quadratic expression enables us to factorise the whole expression using the difference of two squares rule.

WORKED Example 12

Factorise each of the following by first completing the square.

a $x^2 - 8x + 5$ **b** $x^2 + 5x + 1$

THINK	WRITE
a 1 Write the expression.	**a** $x^2 - 8x + 5$
2 Identify the coefficient of x, halve it and square the result.	$= x^2 - 8x + \left(\frac{1}{2} \times -8\right)^2 - \left(\frac{1}{2} \times -8\right)^2 + 5$
3 Add the result of step 2 to the expression, placing it after the x-term. To balance the expression we need to subtract the same amount as we have added so that we have an equivalent expression.	$= x^2 - 8x + (-4)^2 - (-4)^2 + 5$ $= x^2 - 8x + 16 - 16 + 5$
4 Insert brackets around the first three terms to group them and simplify the remaining terms.	$= (x^2 - 8x + 16) - 11$
5 Factorise the first three terms to produce a perfect square.	$= (x - 4)^2 - 11$
6 Rewrite the expression as the difference of two squares.	$= (x - 4)^2 - (\sqrt{11})^2$
7 Factorise using the difference of two squares rule.	$= (x - 4 + \sqrt{11})(x - 4 - \sqrt{11})$
b 1 Write the expression.	**b** $x^2 + 5x + 1$
2 Identify the coefficient of x, halve it and square the result.	$= x^2 + 5x + \left(\frac{1}{2} \times 5\right)^2 - \left(\frac{1}{2} \times 5\right)^2 + 1$
3 Add the result of step 2 to the expression, placing it after the x-term. To balance the expression we need to subtract the same amount as we have added so that we have an equivalent expression.	$= x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 1$ $= x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 1$
4 Insert brackets around the first three terms to group them and simplify the remaining terms. (Find the LCD and add as fractions.)	$= \left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{4}{4}$ $= \left(x^2 + 5x + \frac{25}{4}\right) - \frac{21}{4}$
5 Factorise the first three terms to produce a perfect square.	$= \left(x + \frac{5}{2}\right)^2 - \frac{21}{4}$
6 Rewrite the expression as the difference of two squares.	$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{\sqrt{21}}{2}\right)^2$
7 Factorise using the difference of two squares rule.	$= \left(x + \frac{5}{2} + \frac{\sqrt{21}}{2}\right)\left(x + \frac{5}{2} - \frac{\sqrt{21}}{2}\right)$ or $\left(x + \frac{5 + \sqrt{21}}{2}\right)\left(x + \frac{5 - \sqrt{21}}{2}\right)$

Remember that you can expand the brackets to check your answer.

You may notice that in step 5 of worked example 12a and b the expressions resemble the turning point form. This method will be used to change an expression to turning point form in order to find the turning point of a quadratic graph in the next chapter.

remember

1. If a quadratic trinomial cannot be factorised by finding an integer factor pair, then factorise using the completing the square method:
   (a) take out a common factor and write it outside the brackets if applicable
   (b) halve the value of the coefficient of the x-term and square the result
   (c) add this number to the expression, writing it after the x-term. Balance the expression by making the necessary subtraction.
   (d) factorise the first three terms as a perfect square and simplify the last two terms
   (e) rewrite the expression as the difference of two squares
   (f) factorise using the difference of two squares rule.
2. All factorisations can be checked by expanding.

EXERCISE 4D Factorising by completing the square

WORKED Example 11

1 Complete the square for each of the following expressions.

a $x^2 + 10x$ **b** $x^2 + 6x$ **c** $x^2 - 4x$
d $x^2 + 16x$ **e** $x^2 - 20x$ **f** $x^2 + 8x$
g $x^2 - 14x$ **h** $x^2 + 50x$ **i** $x^2 - 2x$

WORKED Example 12a

Mathcad: Factorising by completing the square

2 Factorise each of the following by first completing the square.

a $x^2 - 4x - 7$ **b** $x^2 + 2x - 2$ **c** $x^2 - 10x + 12$
d $x^2 + 6x - 10$ **e** $x^2 + 16x - 1$ **f** $x^2 - 14x + 43$
g $x^2 + 8x + 9$ **h** $x^2 - 4x - 13$ **i** $x^2 - 12x + 25$

3 Factorise each of the following by first looking for a common factor and then completing the square.

a $2x^2 + 4x - 4$ **b** $4x^2 - 8x - 20$ **c** $5x^2 + 30x + 5$
d $3x^2 - 12x - 39$ **e** $5x^2 - 30x + 10$ **f** $6x^2 + 24x - 6$
g $3x^2 + 30x + 39$ **h** $2x^2 - 8x - 14$ **i** $6x^2 + 36x - 30$

WORKED Example 12b

4 Factorise each of the following by first completing the square.

a $x^2 - x - 1$ **b** $x^2 - 3x - 3$ **c** $x^2 + x - 5$
d $x^2 + 3x - 1$ **e** $x^2 + 5x + 2$ **f** $x^2 + 5x - 2$
g $x^2 - 7x - 1$ **h** $x^2 - 9x + 13$ **i** $x^2 - x - 3$

5 **multiple choice**

a To complete the square, the term which should be added to $x^2 + 4x$ is:
A 16 **B** 4 **C** $4x$ **D** 2 **E** $2x$

b To factorise the expression $x^2 - 3x + 1$, the term that must be both added and subtracted is:
A 9 **B** 3 **C** $3x$ **D** $\frac{3}{2}$ **E** $\frac{9}{4}$

10 Quick Questions 1

1. Expand and simplify $4(x-1)(2x-3)$.
2. Expand $-7(x+3)^2$.
3. Expand $(2x-7)(2x+7)$.
4. Factorise $24x^3 - 18x$.
5. Factorise $98x^2 - 72y^2$.
6. Factorise $4x^2 - 8x + xy - 2y$.
7. Factorise $x^2 - 9x - 22$.
8. Factorise $6x^2 + 19x + 15$.
9. Factorise $4x^2 - 26x - 14$.
10. Factorise $x^2 + 6x - 19$ by completing the square.

Mixed factorisation

The following exercise will provide you with practice in recognising the appropriate method of factorising needed for each expression.

EXERCISE 4E Mixed factorisation

SkillSHEET 4.6 Factorising by grouping three and one

Mathcad Factorising

Factorise each of the following expressions in questions 1–45.

1. $3x + 9$
2. $x^2 + 4x + 4 - 9y^2$
3. $x^2 - 36$
4. $x^2 - 49$
5. $5x^2 - 9x - 2$
6. $15x - 20y$
7. $5c + de + dc + 5e$
8. $5x^2 - 80$
9. $-x^2 - 6x - 5$
10. $x^2 + x - 12$
11. $mn + 1 + m + n$
12. $x^2 - 7$
13. $16x^2 - 4x$
14. $5x^2 + 60x + 100$
15. $18 + 9x - 6y - 3xy$
16. $x^2 - 8x + 16 - y^2$
17. $4x^2 + 8$
18. $fg + 2h + 2g + fh$
19. $x^2 - 5$
20. $10mn - 5n + 10m - 5$
21. $x^2 + 6x + 5$
22. $x^2 - 10x - 11$
23. $x^2 - 4$
24. $-5a + bc + ac - 5b$
25. $xy - 1 + x - y$
26. $3x^2 + 5x + 2$
27. $7x^2 - 28$
28. $-4x^2 - 28x - 24$
29. $2p - rs + pr - 2s$
30. $3x^2 - 27$
31. $-3u + tv + ut - 3v$
32. $x^2 - 11$
33. $12x^2 - 7x + 1$
34. $(x-1)^2 - 4$
35. $(x+2)^2 - 16$
36. $(2x+3)^2 - 25$
37. $3(x+5)^2 - 27$
38. $25 - (x-2)^2$
39. $4(3-x)^2 - 16y^2$
40. $(x+2y)^2 - (2x+y)^2$
41. $(x+3)^2 - (x+1)^2$
42. $(2x-3y)^2 - (x-y)^2$
43. $(x+3)^2 + 5(x+3) + 4$
44. $(x-3)^2 + 3(x-3) - 10$
45. $2(x+1)^2 + 5(x+1) + 2$
46. Consider the following product of algebraic fractions.

$$\frac{x^2+3x-10}{x^2-4} \times \frac{x^2+4x+4}{x^2-2x-8}$$

   a. Factorise the expression in each numerator and denominator.
   b. Cancel factors common to both the numerator and the denominator.
   c. Simplify the expression as a single fraction.

47 Use the procedure in question **46** to factorise and simplify each of the following.

a $\dfrac{x^2-4x+3}{x^2-4x-12} \times \dfrac{x^2+5x+6}{x^2-9}$

b $\dfrac{3x^2-17x+10}{6x^2+5x-6} \times \dfrac{x^2-1}{x^2-6x+5}$

c $\dfrac{6x-12}{x^2-4} \times \dfrac{3x+6}{x(x-5)}$

d $\dfrac{6x^2-x-2}{2x^2+3x+1} \times \dfrac{2x^2+x-1}{3x^2+10x-8}$

e $\dfrac{x^2+4x-5}{x^2+x-2} \div \dfrac{x^2+10x+25}{x^2+4x+4}$

f $\dfrac{x^2-7x+6}{x^2+x-2} \div \dfrac{x^2-x-12}{x^2-2x-8}$

g $\dfrac{4ab+8a}{(c-3)} \div \dfrac{5ac+5a}{c^2-2c-3}$

h $\dfrac{p^2-7p}{p^2-49} \div \dfrac{p^2+p-6}{p^2+14p+49}$

i $\dfrac{m^2+4m+4-n^2}{4m^2-4m-15} \div \dfrac{2m^2+4m-2mn}{10m^2+15m}$

j $\dfrac{d^2-6d+9-25e^2}{12d^2+d-6} \div \dfrac{4d-12-20e}{15d-10}$

4.7 SkillSHEET Multiplication of fractions

4.8 SkillSHEET Division of fractions

4.9 SkillSHEET Simplifying algebraic fractions

Solving quadratic equations

The general form of a quadratic equation is $ax^2 + bx + c = 0$. We can solve this equation algebraically to find x by using the **Null Factor Law**.

If $a \times b = 0$ then $a = 0$ or $b = 0$ (or both a and b equal 0).

The Null Factor Law works only for expressions in factor form.

WORKED Example 13

Solve the equation $(x - 7)(x + 11) = 0$.

THINK	WRITE
1 Write the equation and check that the right-hand side equals zero.	$(x-7)(x+11) = 0$
2 The left-hand side is factorised so use the Null Factor Law to find two linear equations.	$x-7=0$ or $x+11=0$
3 Solve for x.	$x=7$ $\quad x=-11$

Equations that are not in factor form will need to be factorised first before the Null Factor Law can be applied. Remember that the right-hand side of the equation must be zero.

WORKED Example 14

Solve each of the following equations.

a $x^2 - 3x = 0$ **b** $3x^2 - 27 = 0$ **c** $x^2 - 13x + 42 = 0$ **d** $36x^2 - 21x = 2$

THINK	WRITE
a 1 Write the equation. Check that the right-hand side equals zero.	**a** $x^2 - 3x = 0$
2 Take out any common factors (x).	$x(x-3) = 0$
3 Use the Null Factor Law to write two linear equations.	$x = 0$ or $x - 3 = 0$
4 Solve for x.	$x = 3$

Continued over page

THINK	WRITE
b 1 Write the equation. Check that the right-hand side equals zero.	**b** $3x^2 - 27 = 0$
2 Take out any common factors (3).	$3(x^2 - 9) = 0$
3 Look at the number of terms to factorise and identify the appropriate method. Factorise using the difference of two squares rule.	$3(x^2 - 3^2) = 0$ $3(x + 3)(x - 3) = 0$
4 Use the Null Factor Law to write two linear equations.	$x + 3 = 0$ or $x - 3 = 0$
5 Solve for x.	$x = -3 \qquad x = 3$
c 1 Write the equation. Check that the right-hand side equals zero. Check for any common factors (none).	**c** $x^2 - 13x + 42 = 0$
2 Look at the number of terms to factorise and identify the appropriate method. Factorise by finding a factor pair of 42 that adds to −13.	$42: -6 + -7 = -13$ $(x - 6)(x - 7) = 0$
3 Use the Null Factor Law to write two linear equations.	$x - 6 = 0$ or $x - 7 = 0$
4 Solve for x.	$x = 6 \qquad x = 7$
d 1 Write the equation. Check that the right-hand side equals zero. (It does not.)	**d** $36x^2 - 21x = 2$
2 Rearrange the equation so the right-hand side of the equation equals zero. Check for any common factors (none).	$36x^2 - 21x - 2 = 0$
3 Recognise that the expression to factorise is a quadratic trinomial. Identify a factor pair of ac (−72) which adds to the coefficient of x (−21).	$-72: 3 + -24 = -21$
4 Rewrite the expression by breaking the x-term into two terms using the factor pair from step 3.	$36x^2 - 24x + 3x - 2 = 0$
5 Factorise the expression by grouping.	$12x(3x - 2) + (3x - 2) = 0$ $(3x - 2)(12x + 1) = 0$
6 Use the Null Factor Law to write two linear equations.	$3x - 2 = 0$ or $12x + 1 = 0$ $3x = 2 \qquad 12x = -1$
7 Solve for x.	$x = \frac{2}{3} \qquad x = \frac{-1}{12}$

Solving quadratic equations by completing the square

If it is not possible to find an integer factor pair when factorising a quadratic trinomial, the completing the square method can be used before applying the Null Factor Law to the equation. This method allows us to find irrational solutions or, in other words, the solutions will be surds.

WORKED Example 15

Find the solutions to the equation $x^2 + 2x - 4 = 0$. Give exact answers.

THINK	WRITE
1 Write the equation. Check that the right-hand side equals zero.	$x^2 + 2x - 4 = 0$
2 Identify the coefficient of x, halve it and square the result.	$x^2 + 2x + \left(\frac{1}{2} \times 2\right)^2 - \left(\frac{1}{2} \times 2\right)^2 - 4 = 0$
3 Add the result of step 2 to the equation, placing it after the x-term. To balance the equation, we need to subtract the same amount as we have added so that we have an equivalent equation.	$x^2 + x + (1)^2 - (1)^2 - 4 = 0$ $x^2 + 2x + 1 - 1 - 4 = 0$
4 Insert brackets around the first three terms to group them and simplify the remaining terms.	$(x^2 + 2x + 1) - 5 = 0$
5 Factorise the first three terms to produce a perfect square.	$(x + 1)^2 - 5 = 0$
6 Write as the difference of two squares and factorise.	$(x + 1)^2 - (\sqrt{5})^2 = 0$ $(x + 1 + \sqrt{5})(x + 1 - \sqrt{5}) = 0$
7 Use the Null Factor Law to find linear equations.	$x + 1 + \sqrt{5} = 0$ or $x + 1 - \sqrt{5} = 0$
8 Solve for x. Keep the answer in surd form to provide an exact answer.	$x = -1 - \sqrt{5}$ $\quad x = -1 + \sqrt{5}$ This can be written as $x = -1 \pm \sqrt{5}$.

There are many problems that can be modelled by the solution to a quadratic equation. First you may need to form the quadratic equation which represents the situation.

WORKED Example 16

When two consecutive numbers are multiplied, the result is 20. Find the numbers.

THINK	WRITE
1 Define the terms by using a pronumeral for one of the numbers and adding 1 to it to give the second number.	Let the two numbers be x and $x + 1$.
2 Write an equation multiplying the numbers to give the answer.	$x(x + 1) = 20$
3 Rearrange the equation so that the right-hand side equals zero.	$x(x + 1) - 20 = 0$
4 Expand to remove the brackets.	$x^2 + x - 20 = 0$

Continued over page

THINK	WRITE
5 Factorise.	$(x + 5)(x - 4) = 0$
6 Use the Null Factor Law to solve for x.	$x + 5 = 0$ or $x - 4 = 0$ $x = -5 \qquad x = 4$
7 Use the answer to find the other number.	If $x = -5$, $x + 1 = -4$. If $x = 4$, $x + 1 = 5$.
8 Answer the question in a sentence.	The numbers are 4 and 5 or -5 and -4.
9 Check the solutions.	Check: $4 \times 5 = 20 \quad -5 \times -4 = 20$

remember

1. The general form of a quadratic equation is $ax^2 + bx + c = 0$.
2. To solve a quadratic equation:
   (a) make sure that the right-hand side of the equation equals zero
   (b) take out any common factors
   (c) factorise the left-hand side if applicable
   (d) use the Null Factor Law to solve for x.
3. An exact answer is a surd or an answer that has not been rounded or approximated.

GC program—TI: Solving quadratic equations

GC program—Casio: Solving quadratic equations

EXERCISE 4F Solving quadratic equations

Mathcad: Solving quadratic equations

EXCEL Spreadsheet: Solving $ax^2 + bx = 0$

EXCEL Spreadsheet: Solving $ax^2 + c = 0$

EXCEL Spreadsheet: Solving $x^2 + bx + c = 0$

WORKED Example 13

1 Solve each of the following equations.
- **a** $(x + 7)(x - 9) = 0$
- **b** $(x - 3)(x + 2) = 0$
- **c** $(x - 2)(x - 3) = 0$
- **d** $x(x - 3) = 0$
- **e** $x(x - 1) = 0$
- **f** $x(x + 5) = 0$
- **g** $2x(x - 3) = 0$
- **h** $9x(x + 2) = 0$
- **i** $(x - \frac{1}{2})(x + \frac{1}{2}) = 0$
- **j** $-(x + 1.2)(x + 0.5) = 0$
- **k** $2(x - 0.1)(2x - 1.5) = 0$
- **l** $(x + \sqrt{2})(x - \sqrt{3}) = 0$

2 Solve each of the following equations.
- **a** $(2x - 1)(x - 1) = 0$
- **b** $(3x + 2)(x + 2) = 0$
- **c** $(4x - 1)(x - 7) = 0$
- **d** $(7x + 6)(2x - 3) = 0$
- **e** $(5x - 3)(3x - 2) = 0$
- **f** $(8x + 5)(3x - 2) = 0$
- **g** $x(x - 3)(2x - 1) = 0$
- **h** $x(2x - 1)(5x + 2) = 0$
- **i** $x(x + 3)(5x - 2) = 0$

WORKED Example 14a

3 Solve each of the following equations.
- **a** $x^2 - 2x = 0$
- **b** $x^2 + 5x = 0$
- **c** $x^2 = 7x$
- **d** $3x^2 = -2x$
- **e** $4x^2 - 6x = 0$
- **f** $6x^2 - 2x = 0$
- **g** $4x^2 - 2\sqrt{7}x = 0$
- **h** $3x^2 + \sqrt{3}x = 0$
- **i** $15x - 12x^2 = 0$

WORKED Example 14b

4 Solve each of the following equations.
- **a** $x^2 - 4 = 0$
- **b** $x^2 - 25 = 0$
- **c** $3x^2 - 12 = 0$
- **d** $4x^2 - 196 = 0$
- **e** $9x^2 - 16 = 0$
- **f** $4x^2 - 25 = 0$
- **g** $9x^2 = 4$
- **h** $36x^2 = 9$
- **i** $x^2 - \frac{1}{25} = 0$
- **j** $\frac{1}{36}x^2 - \frac{4}{9} = 0$
- **k** $x^2 - 5 = 0$
- **l** $9x^2 - 11 = 0$

WORKED Example 14c

5 Solve each of the following equations.
- **a** $x^2 - x - 6 = 0$
- **b** $x^2 + 6x + 8 = 0$
- **c** $x^2 - 6x - 7 = 0$
- **d** $x^2 - 8x + 15 = 0$
- **e** $x^2 - 2x + 1 = 0$
- **f** $x^2 - 3x - 4 = 0$
- **g** $x^2 - 10x + 25 = 0$
- **h** $x^2 - 3x - 10 = 0$
- **i** $x^2 - 8x + 12 = 0$
- **j** $x^2 - 4x - 21 = 0$
- **k** $x^2 - x - 30 = 0$
- **l** $x^2 - 7x + 12 = 0$

WORKED Example 14d

6 Solve each of the following equations.

a $2x^2 - 5x = 3$ b $3x^2 + x - 2 = 0$ c $5x^2 + 9x = 2$
d $6x^2 - 11x + 3 = 0$ e $14x^2 - 11x = 3$ f $12x^2 - 7x + 1 = 0$
g $6x^2 - 7x = 20$ h $12x^2 + 37x + 28 = 0$ i $10x^2 - x = 2$
j $6x^2 - 25x + 24 = 0$ k $30x^2 + 7x - 2 = 0$ l $3x^2 - 21x = -36$

EXCEL Spreadsheet Solving $ax^2 + bx + c = 0$

WORKED Example 15

7 Find the solutions for each of the following equations. Give exact answers.

a $x^2 - 4x + 2 = 0$ b $x^2 + 2x - 2 = 0$ c $x^2 + 6x - 1 = 0$
d $x^2 - 8x + 4 = 0$ e $x^2 - 10x + 1 = 0$ f $x^2 - 2x - 2 = 0$
g $x^2 + 2x - 5 = 0$ h $x^2 + 4x - 6 = 0$ i $x^2 + 4x - 11 = 0$

8 Find the solutions for each of the following equations. Give exact answers.

a $x^2 - 3x + 1 = 0$ b $x^2 + 5x - 1 = 0$ c $x^2 - 7x + 4 = 0$
d $x^2 - 5 = x$ e $x^2 - 11x + 1 = 0$ f $x^2 + x = 1$
g $x^2 + 3x - 7 = 0$ h $x^2 - 3 = 5x$ i $x^2 - 9x + 4 = 0$

GC program—TI Completing the square

9 Solve each of the following equations, rounding answers to 2 decimal places.

a $2x^2 + 4x - 6 = 0$ b $3x^2 + 12x - 3 = 0$ c $5x^2 - 10x - 15 = 0$
d $4x^2 - 8x - 8 = 0$ e $2x^2 - 6x + 2 = 0$ f $3x^2 - 9x - 3 = 0$
g $5x^2 - 15x - 25 = 0$ h $7x^2 + 7x - 21 = 0$ i $4x^2 + 8x - 2 = 0$

GC program—Casio Completing the square

10 Are there real solutions to the equation $x^2 + 4x + 10 = 0$? Give reasons for your answer.

WORKED Example 16

11 When two consecutive numbers are multiplied, the result is 72. Find the numbers.

12 When two consecutive even numbers are multiplied, the result is 48. Find the numbers.

13 When a number is added to its square the result is 90. Find the number.

14 Twice a number is added to three times its square. If the result is 16, find the number.

15 Five times a number is added to two times its square. If the result is 168, find the number.

16 A soccer ball is kicked. The height, h, in metres, of the soccer ball t seconds after it was kicked can be represented by the equation $h = -t(t - 6)$. Find how long it takes for the soccer ball to hit the ground again.

17 The length of an Australian flag is twice its width and the diagonal length is 45 cm.

a If x cm is the width of the flag, find the length in terms of x.

b Draw a diagram of the flag marking in the diagonal. Mark the length and the width in terms of x.

c Use Pythagoras' theorem to write an equation relating the lengths of the sides to the length of the diagonal.

d Solve the equation to find the dimensions of the Australian flag. Round the answer to the nearest cm.

18 If the length of a paddock is 2 metres more than its width and the area is 48 m^2, find the length and width of the paddock.

19 Henrietta is a pet rabbit who lives in an enclosure, which is 2 m wide and 4 m long. Her human family has decided to purchase some more rabbits to keep her company so the size of the enclosure must be increased.

a Draw a diagram of Henrietta's enclosure, clearly marking the lengths of the sides.

b If the length and width of the enclosure are increased by x m, find the new dimensions.

c If the new area is to be 24 m^2, write an equation relating the sides and the area of the enclosure (area = length × width).

d Use the equation to find the value of x and, hence, the length of the sides of the new enclosure.

20 A student is required to cover an area of 620 cm^2 with mosaic tiles. The tile pattern is to be surrounded by a border 2 cm wide to complete the display page.

The length of the display page is l cm and its width is 4 cm less than its length.

a Find the width of the display page in terms of l.

b Find the width and length of the tile pattern in terms of l.

c Using the answers from **b** write an equation relating the area of the tile pattern to its dimensions.

d Use the method of completing the square to solve the equation and, hence, find the length, l cm, of the display page. Round the answer to the nearest cm.

e Find the area of the display page. Round the answer to the nearest cm^2.

GAME time
Quadratic equations — 002

WorkSHEET 4.2

Using the quadratic formula

The method of solving quadratic equations by completing the square can be generalised to produce what is called the quadratic formula. Consider solving the general equation $ax^2 + bx + c = 0$. We will follow the steps involved in completing the square.

1. Divide both sides of the equation by a. $$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
2. Complete the square. $$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$
3. Factorise the first three terms as a perfect square. $$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$
4. Add the final two terms. $$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$$
5. Write as the difference of two squares. $$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2 = 0$$
6. Factorise using the difference of two squares rule. $$\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right) = 0$$
7. Solve the two linear factors. $$x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} = 0 \quad \text{or} \quad x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} = 0$$ $$x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$

The solution can be summarised as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where a is the coefficient of x^2, b is the coefficient of x and c is the constant or the term without an x. This formula can be used to solve any quadratic equation.

WORKED Example 17

Use the quadratic formula to solve each of the following equations.

a $3x^2 + 4x + 1 = 0$ **(exact answer)**

b $-3x^2 - 6x - 1 = 0$ **(round to 2 decimal places)**

THINK	WRITE
a 1 Write the equation.	**a** $3x^2 + 4x + 1 = 0$
2 Write the quadratic formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3 State the values for a, b and c.	where $a = 3$, $b = 4$, $c = 1$
4 Substitute the values in the formula.	$x = \dfrac{-4 \pm \sqrt{16 - 4 \times 3 \times 1}}{2 \times 3}$
5 Simplify and solve for x.	$= \dfrac{-4 \pm \sqrt{4}}{6}$
	$= \dfrac{-4 \pm 2}{6}$
	$x = \dfrac{-4 + 2}{6}$ or $x = \dfrac{-4 - 2}{6}$
	$x = -\dfrac{1}{3}$ $\quad$ $x = -1$

Continued over page

THINK	WRITE
b 1 Write the equation.	**b** $-3x^2 - 6x - 1 = 0$
2 Write the quadratic formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3 State the values for a, b and c.	where $a = -3$, $b = -6$, $c = -1$
4 Substitute the values in the formula.	$x = \dfrac{-(-6) \pm \sqrt{36 - 4 \times -3 \times -1}}{2 \times -3}$
5 Simplify the fraction.	$= \dfrac{6 \pm \sqrt{24}}{-6}$ $= \dfrac{6 \pm 2\sqrt{6}}{-6}$ $= \dfrac{3 \pm \sqrt{6}}{-3}$ $x = \dfrac{3 + \sqrt{6}}{-3}$ or $\dfrac{3 - \sqrt{6}}{-3}$
6 Solve for x.	$x \approx -1.82$ or $x \approx -0.18$

Note that when asked to give an answer in exact form you may obtain a surd answer which may need to be simplified.

remember

The quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to solve quadratic equations of the form $ax^2 + bx + c = 0$.

EXERCISE 4G Using the quadratic formula

SkillSHEET 4.10 Substituting into the quadratic formula

1 State the values for a, b and c in each of the following equations of the form $ax^2 + bx + c = 0$.

a $3x^2 - 4x + 1 = 0$ **b** $7x^2 - 12x + 2 = 0$ **c** $8x^2 - x - 3 = 0$
d $x^2 - 5x + 7 = 0$ **e** $5x^2 - 5x - 1 = 0$ **f** $4x^2 - 9x - 3 = 0$
g $12x^2 - 29x + 103 = 0$ **h** $43x^2 - 81x - 24 = 0$ **i** $6x^2 - 15x + 1 = 0$

SkillSHEET 4.11 Simplifying surds

WORKED Example 17a

2 Use the quadratic formula to solve each of the following equations. Give exact answers.

a $x^2 + 2x + 1 = 0$ **b** $x^2 + 3x - 1 = 0$ **c** $x^2 - 5x + 2 = 0$
d $x^2 - 4x - 9 = 0$ **e** $x^2 + 2x - 11 = 0$ **f** $x^2 - 7x + 1 = 0$
g $x^2 - 9x + 2 = 0$ **h** $x^2 - 6x - 3 = 0$ **i** $x^2 + 8x - 15 = 0$
j $-x^2 + x + 5 = 0$ **k** $-x^2 + 5x + 2 = 0$ **l** $-x^2 - 2x + 7 = 0$

Mathcad Quadratic formula

WORKED Example 17b

3 Use the quadratic formula to solve each of the following equations. Give approximate answers rounded to 2 decimal places.

a $3x^2 - 4x - 3 = 0$ **b** $4x^2 - x - 7 = 0$ **c** $2x^2 + 7x - 5 = 0$
d $7x^2 + x - 2 = 0$ **e** $5x^2 - 8x + 1 = 0$ **f** $2x^2 - 13x + 2 = 0$

EXCEL Spreadsheet Quadratic formula

g $-3x^2 + 2x + 7 = 0$ **h** $-7x^2 + x + 8 = 0$ **i** $-12x^2 + x + 9 = 0$
j $-6x^2 + 4x + 5 = 0$ **k** $-11x^2 - x + 1 = 0$ **l** $-4x^2 - x + 7 = 0$
m $-2x^2 + 12x - 1 = 0$ **n** $-5x^2 + x + 3 = 0$ **o** $-3x^2 + 5x + 2 = 0$

4 **multiple choice**
The solutions of the equation $3x^2 - 7x - 2 = 0$ are:
A 1, 2 **B** 1, −2 **C** −0.257, 2.59
D −0.772, 7.772 **E** −1.544, 15.544

5 Solve each of the following equations using any suitable method. Round to 3 decimal places where appropriate.
a $2x^2 - 7x + 3 = 0$ **b** $x^2 - 5x = 0$ **c** $x^2 - 2x - 3 = 0$
d $x^2 - 3x + 1 = 0$ **e** $x^2 - 7x + 2 = 0$ **f** $x^2 - 6x + 8 = 0$
g $x^2 - 5x + 8 = 0$ **h** $x^2 - 7x - 8 = 0$ **i** $x^2 + 2x - 9 = 0$
j $3x^2 + 3x - 6 = 0$ **k** $2x^2 + 11x - 21 = 0$ **l** $7x^2 - 2x + 1 = 0$
m $-x^2 + 9x - 14 = 0$ **n** $-6x^2 - x + 1 = 0$ **o** $-6x^2 + x - 5 = 0$

6 The surface area of a closed cylinder is given by the formula SA = $2\pi r(r + h)$, where r cm is the radius of the can and h cm is the height.
The height of a can of wood finish is 7 cm and its surface area is 231 cm^2.
a Substitute values into the formula to form a quadratic equation using the pronumeral, r.
b Use the quadratic formula to solve the equation and, hence, find the radius of the can. Round the answer to 1 decimal place.
c Calculate the area of the paper label on the can. Round the answer to the nearest square centimetre.

10 QUICK QUESTIONS 2

1 Solve the quadratic equation $(x + 7)(x + 2) = 0$.

2 Solve the equation $x^2 - 9x - 36 = 0$ by writing the equation as the product of linear factors.

3 Solve the equation $-3x^2 - 5x = 0$.

4 Solve the equation $x^2 - 121 = 0$.

5 Solve the equation $12x^2 - 11x - 15 = 0$.

6 Solve the equation $2x^2 = 36$.

7 Solve the equation $x^2 - 4x - 6 = 0$ by completing the square, leaving your answer in exact form.

8 Solve the equation $x^2 - x - 3 = 0$ by completing the square, leaving your answer in exact form.

9 Solve the equation $4x^2 - 8x + 3 = 0$ by using the quadratic formula.

10 Solve the equation $x^2 + x - 7 = 0$ giving your answer correct to 3 decimal places.

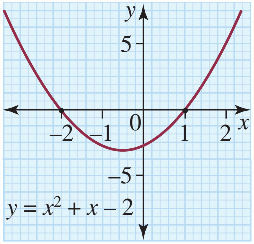
Finding solutions to quadratic equations by inspecting graphs

We saw in the previous two sections that a quadratic equation written in standard form has solutions when the graph of $y = ax^2 + bx + c$ is equal to zero. In this section we will find solutions or *roots* of quadratic equations by inspecting their corresponding graphs.

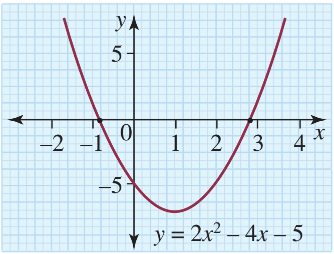
WORKED Example 18

Determine the solution (or roots) of each of the following quadratic equations by inspecting their corresponding graphs. Round answers to 1 decimal place where appropriate.

a $x^2 + x - 2 = 0$



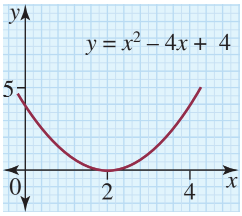
b $2x^2 - 4x - 5 = 0$



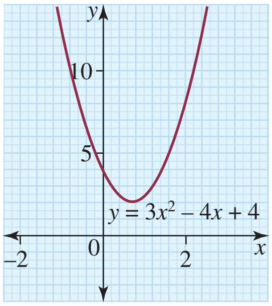
THINK	WRITE
a The corresponding graph of $y = x^2 + x - 2$ is equal to zero when $y = 0$. Look at the graph to find where $y = 0$; that is, where it crosses the x-axis.	**a** $x^2 + x - 2 = 0$ From the graph, the solutions are $x = 1$ and $x = -2$.
b The corresponding graph of $y = 2x^2 - 4x - 5$ is equal to zero when $y = 0$. Look at the graph to see where $y = 0$; that is, where it crosses the x-axis. We can only find estimates of the solutions.	**b** $2x^2 - 4x - 5 = 0$ From the graph, the solutions are approximately $x = -0.8$ and $x = 2.8$.

Note: There are some quadratic equations that have only one solution.

For example, $x^2 - 4x + 4 = 0$ has the one solution of $x = 2$. The graph of $y = x^2 - 4x + 4$ touches the x-axis only at $x = 2$.



There are also other quadratic equations that have no real solutions. For example, the graph of $y = 3x^2 - 4x + 4$ does not touch or cross the x-axis and so $3x^2 - 4x + 4 = 0$ has no real solutions (that is, no solutions that are real numbers).



Confirming solutions

It is possible to confirm the solutions obtained by looking at the graphs. As we saw with linear equations, this is achieved by substituting the solution or solutions into the original quadratic equation. If both sides of the equation are equal, the solution is correct.

WORKED Example 19

Confirm, by substitution, the solutions obtained in worked example 18.

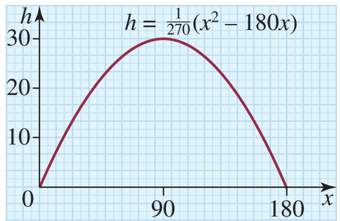
a $x^2 + x - 2 = 0$; **solutions:** $x = 1$ **and** $x = -2$

b $2x^2 - 4x - 5 = 0$; **solutions:** $x = -0.8$ **and** $x = 2.8$

THINK	WRITE
a 1 Write the left-hand side of the equation and substitute $x = 1$ into the expression.	**a** When $x = 1$, $x^2 + x - 2 = 1^2 + 1 - 2$
2 Simplify to check that the expression is equal to zero.	$= 0$ Solution is confirmed.
3 Write the expression and substitute $x = -2$.	When $x = -2$, $x^2 + x - 2 = (-2)^2 + -2 - 2$
4 Simplify to check that the expression is equal to zero.	$= 4 - 2 - 2$ $= 0$ Solution is confirmed.
b 1 Write the left-hand side of the equation and substitute $x = -0.8$ into the expression.	**b** When $x = -0.8$, $2x^2 - 4x - 5 = 2 \times (-0.8)^2 - 4 \times -0.8 - 5$
2 Simplify to check that the expression is close to zero.	$= 1.28 + 3.2 - 5$ $= -0.52$ As -0.8 is only an estimate, the left-hand side expression can be said to be close to zero.
3 Write the expression and substitute $x = 2.8$ into the expression.	When $x = 2.8$, $2x^2 - 4x - 5 = 2 \times (2.8)^2 - 4 \times 2.8 - 5$
4 Simplify to check that the expression is close to zero.	$= 15.68 - 11.2 - 5$ $= -0.52$ As 2.8 is only an estimate, the left-hand side expression can be said to be close to zero.

WORKED Example 20

A golf ball hit along a fairway follows the path shown in the graph.



The height, h metres after it has travelled x metres horizontally, follows the rule $h = -\frac{1}{270}(x^2 - 180x)$.
Use the graph to find how far the ball landed from the golfer.

THINK

On the graph, the ground is represented by the x-axis since this is where $h = 0$. The golf ball is on the ground when the graph intersects with the x-axis.

WRITE

The golf ball lands 180 m from the golfer.

remember

1. The solution(s) or root(s) of any equation can be found by inspecting the corresponding graph.
2. The root of any graph is the x-intercept or the x-coordinate of the point where the graph crosses the x-axis.
3. The roots or intercepts of the quadratic graph $y = ax^2 + bx + c$ are the solutions to the equation $ax^2 + bx + c = 0$.

EXERCISE 4H Finding solutions to quadratic equations by inspecting graphs

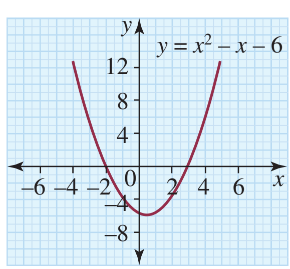
EXCEL Spreadsheet: Graphing quadratics

Mathcad: Inspecting quadratic graphs

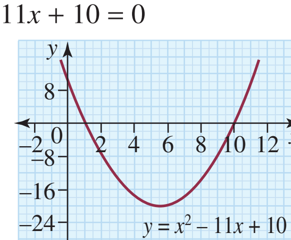
WORKED Example 18

1 Determine the solution (or roots) of each of the following quadratic equations by inspecting the corresponding graphs. Round answers to 1 decimal place where appropriate.

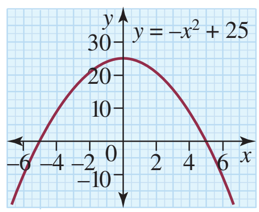
a $x^2 - x - 6 = 0$



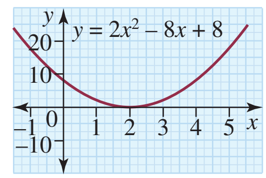
b $x^2 - 11x + 10 = 0$



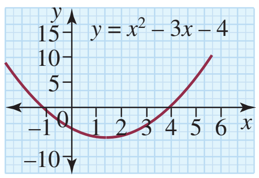
c $-x^2 + 25 = 0$



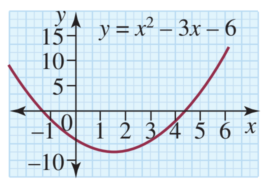
d $2x^2 - 8x + 8 = 0$



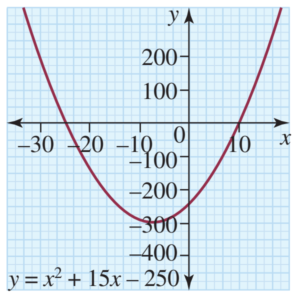
e $x^2 - 3x - 4 = 0$



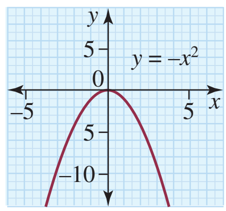
f $x^2 - 3x - 6 = 0$



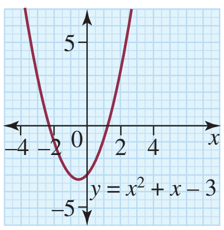
g $x^2 + 15x - 250 = 0$



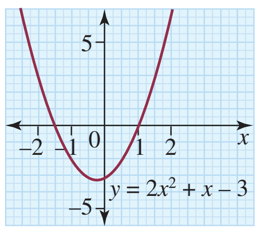
h $-x^2 = 0$



i $x^2 + x - 3 = 0$



j $2x^2 + x - 3 = 0$



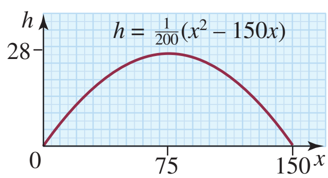
WORKED Example 19

2 Confirm, by substitution, the solutions obtained in question **1**.

WORKED Example 20

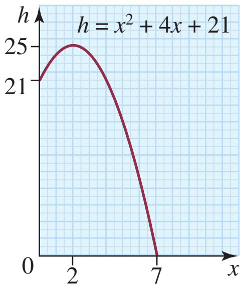
3 A golf ball hit along a fairway follows the path shown in the graph.

The height, h metres after it has travelled x metres horizontally, follows the rule $h = -\frac{1}{200}(x^2 - 150x)$. Use the graph to find how far the ball landed from the golfer.



4 A ball is thrown upwards from a building and follows the path shown in the graph until it lands on the ground.

The ball is h metres above the ground when it is a horizontal distance of x metres from the building. The path of the ball follows the rule $h = -x^2 + 4x + 21$. Use the graph to find how far from the building the ball lands.



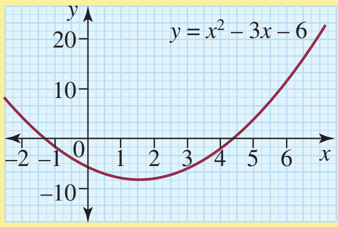
THINKING

Finding solutions to quadratics by interpolation

Consider the quadratic equation $x^2 - 3x - 6 = 0$.

Sketching the corresponding graph, $y = x^2 - 3x - 6$, using a graphics calculator or a computer graphing program, show that there is a solution between $x = 4$ and $x = 6$.

This can be confirmed using the following logic:



Step 1 The value of $y = x^2 - 3x - 6$ when $x = 4$, written as $y(4) = 4^2 - 3 \times 4 - 6 = -2$.
The value of $y = x^2 - 3x - 6$ when $x = 6$, written as $y(6) = 6^2 - 3 \times 6 - 6 = 12$.
Since the graph moves from *below* the x-axis at $x = 4$, to *above* the x-axis at $x = 6$, it is reasonable to assume that there is a solution somewhere *between* $x = 4$ and $x = 6$.

Step 2 Choose a value between $x = 4$ and $x = 6$, say 5, for example.
The value of $y = x^2 - 3x - 6$ when $x = 4$, written as $y(4) = 4^2 - 3 \times 4 - 6 = -2$.
The value of $y = x^2 - 3x - 6$ when $x = 5$, written as $y(5) = 5^2 - 3 \times 5 - 6 = 4$.
The value of $y = x^2 - 3x - 6$ when $x = 6$, written as $y(6) = 6^2 - 3 \times 6 - 6 = 12$.
Since the graph moves from *below* the x-axis at $x = 4$, to *above* the x-axis at $x = 5$, it is reasonable to assume that there is a solution somewhere between $x = 4$ and $x = 5$.

Step 3 Choose a value between $x = 4$ and $x = 5$, say 4.5, for example.
The value of $y = x^2 - 3x - 6$ when $x = 4$: $y(4) = 4^2 - 3 \times 4 - 6 = -2$.
The value of $y = x^2 - 3x - 6$ when $x = 4.5$: $y(4.5) = 4.5^2 - 3 \times 4.5 - 6 = 0.75$.
The value of $y = x^2 - 3x - 6$ when $x = 5$: $y(5) = 5^2 - 3 \times 5 - 6 = 4$.
Since the graph moves from *below* the x-axis at $x = 4$, to *above* the x-axis at $x = 4.5$, it is reasonable to assume that there is a solution somewhere between $x = 4$ and $x = 4.5$.

1 Repeat step 3 several times and check that you get closer and closer to the solution, which is approximately $x = 4.372$.

2 Repeat the process to find the other root, somewhere between -2 and 0.

3 This process can be done on a spreadsheet. When you are using the CD, click on the Excel spreadsheet icon for instructions on how to do this.

EXCEL Spreadsheet: Finding solutions by interpolation

Using the discriminant

The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives the solutions to the general quadratic equation $ax^2 + bx + c = 0$. By examining the expression under the square root sign, $b^2 - 4ac$, we can determine the number and type of solutions produced and hence the number of x-intercepts when the quadratic equation is graphed.

The expression $b^2 - 4ac$ is known as the **discriminant** (and is denoted by the symbol Δ).

Case 1 $\Delta < 0$

If $x^2 + 2x + 3 = 0$, then $a = 1$, $b = 2$ and $c = 3$.

$$\Delta = b^2 - 4ac \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= 2^2 - 4 \times 1 \times 3 \qquad = \frac{-2 \pm \sqrt{-8}}{2}$$

$$= -8$$

If the discriminant is less than zero, there are no real solutions because the expression under the square root sign is negative. It is not possible to find a real number which is the square root of a negative number.

Hence, the graph of $y = x^2 + 2x + 3$ will not touch or cross the x-axis.

Case 2 $\Delta = 0$

If $4x^2 + 12x + 9 = 0$, then $a = 4$, $b = 12$ and $c = 9$.

$$\Delta = b^2 - 4ac \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= 12^2 - 4 \times 4 \times 9 \qquad = \frac{-12 \pm 0}{2 \times 4}$$

$$= 144 - 144 \qquad = -\tfrac{12}{8}$$

$$= 0 \qquad = -\tfrac{3}{2}$$

If the discriminant is equal to zero then the two solutions are the same. This may be regarded as one rational solution that is equal to $\frac{-b}{2a}$.

That is, if $b^2 - 4ac = 0$, then $x = \frac{-b + 0}{2a}$ which is the same as $x = \frac{-b - 0}{2a}$.

One solution indicates that the quadratic trinomial is a perfect square that can be factorised easily using the perfect squares rule; that is, $4x^2 - 12x + 9 = (2x - 3)^2$.

Hence, the graph of $y = 4x^2 + 12x + 9$ will touch the x-axis once.

Case 3 $\Delta > 0$

If the discriminant is positive, there are two distinct solutions. We can determine more information than this by checking whether the discriminant is also a perfect square.

(a) If $2x^2 - 7x - 4 = 0$, then $a = 2$, $b = -7$ and $c = -4$.

$$\Delta = b^2 - 4ac \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= (-7)^2 - 4 \times 2 \times -4 \qquad = \frac{7 \pm \sqrt{81}}{2 \times 2}$$

$$= 49 + 32 \qquad = \frac{7 \pm 9}{4}$$

$$= 81 \qquad x = 4 \text{ or } x = -\tfrac{1}{2}$$

If the discriminant is positive and a perfect square, the quadratic trinomial will have two rational solutions. This means the quadratic trinomial can be factorised easily; that is, $2x^2 - 7x - 4 = (2x + 1)(x - 4)$.

(b) If $x^2 - 5x - 1 = 0$ then $a = 1$, $b = -5$ and $c = -1$.

$$\Delta = b^2 - 4ac \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= (-5)^2 - 4 \times 1 \times -1 \qquad x = \frac{5 \pm \sqrt{29}}{2 \times 1}$$

$$= 25 + 4 \qquad x = \frac{5 \pm \sqrt{29}}{2}$$

$$= 29$$

If the discriminant is positive but not a perfect square, the factors are irrational and the quadratic formula is used to find the two irrational (surd) solutions.

Hence, the graphs of both equations will each have two x-intercepts.

WORKED Example 21

By using the discriminant, determine whether the following equations have:

(i) two rational solutions

(ii) two irrational solutions

(iii) one rational solution (two equal solutions)

(iv) no real solutions.

a $x^2 - 9x - 10 = 0$ **b** $x^2 - 2x - 14 = 0$ **c** $x^2 - 2x + 14 = 0$ **d** $x^2 + 14x = -49$

THINK	WRITE
a 1 Write the equation.	**a** $x^2 - 9x - 10 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = -9, c = -10$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= (-9)^2 - 4 \times 1 \times -10$ $= 121$
4 Identify the number and type of solutions when $\Delta > 0$ and a perfect square.	The equation has two rational solutions.
b 1 Write the equation.	**b** $x^2 - 2x - 14 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = -2, c = -14$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= (-2)^2 - 4 \times 1 \times -14$ $= 60$
4 Identify the number and type of solutions when $\Delta > 0$ but not a perfect square.	The equation has two irrational solutions.
c 1 Write the equation.	**c** $x^2 - 2x + 14 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = -2, c = 14$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= (-2)^2 - 4 \times 1 \times 14$ $= -52$
4 Identify the number and type of solutions when $\Delta < 0$.	The equation has no real solutions.
d 1 Write the equation, then rewrite it so the right side equals zero.	**d** $x^2 + 14x = -49$ $x^2 + 14x + 49 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = 14, c = 49$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= 14^2 - 4 \times 1 \times 49$ $= 0$
4 Identify the number and types of solutions when $\Delta = 0$.	The equation has 1 rational solution.

The number of solutions of a quadratic equation correspond to the number of x-intercepts obtained when the equation is graphed.

remember

1. The discriminant of a quadratic equation is given by $\Delta = b^2 - 4ac$.
2. If $\Delta < 0$ there are no real solutions to the equation.
3. If $\Delta = 0$ there is only one rational solution (or two equal solutions) to the equation. The equation can be factorised easily.
4. If $\Delta > 0$ there are two distinct solutions to the equation.
   (a) If the discriminant is a perfect square, the solutions are rational and the equation can be factorised easily.
   (b) If the discriminant is not a perfect square the solutions are irrational and the equation can be solved using the quadratic formula or completing the square method.
5. The number of solutions of a quadratic equation correspond to the number of x-intercepts obtained when the equation is graphed.

EXERCISE 4I Using the discriminant

Mathcad
Using the discriminant

1 Find the discriminant for each of the following equations.

a $x^2 - 3x + 5$ **b** $4x^2 - 20x + 25 = 0$ **c** $x^2 + 9x - 22 = 0$
d $9x^2 + 12x + 4$ **e** $x^2 + 3x - 7 = 0$ **f** $25x^2 - 10x + 1 = 0$
g $3x^2 - 2x - 4 = 0$ **h** $2x^2 - 5x + 4 = 0$ **i** $x^2 - 10x + 26 = 0$
j $3x^2 + 5x - 7 = 0$ **k** $2x^2 + 7x - 10 = 0$ **l** $x^2 - 11x + 30 = 0$

WORKED Example 21

2 By using the discriminant, determine whether the equations in question **1** have:
i two rational solutions **ii** two irrational solutions
iii one rational solution (two equal solutions) **iv** no real solutions.

3 With the information gained from the discriminant, use the most efficient method to solve each equation in question **1**. Where appropriate, round answers correct to 3 decimal places.

4 Consider the equation $3x^2 + 2x + 7 = 0$.
a What are the values of a, b and c?
b What is the value of $b^2 - 4ac$?
c How many real solutions, and hence x-intercepts, are there for this equation?

5 Consider the equation $-6x^2 + x + 3 = 0$.
a What are the values of a, b and c?
b What is the value of $b^2 - 4ac$?
c How many real solutions, and hence x-intercepts, are there for this equation?
d With the information gained from the discriminant, use the most efficient method to solve the equation. Give an exact answer.

6 **multiple choice**

The discriminant of the equation $x^2 - 4x - 5 = 0$ is:
A 36 **B** 11 **C** 4 **D** 0 **E** -4

WorkSHEET 4.3

COMMUNICATION

Flying dolphin

At the start of the chapter you were given an equation which modelled the leap of a dolphin as it 'flew' out of the water. The equation is $h = -0.4d^2 + d$, where h is the dolphin's height above water and d is the horizontal distance from its starting point. Both h and d are in metres.

1 How high above the water is the dolphin when it has travelled 2 m horizontally from its starting point?

2 What horizontal distance does the dolphin cover when it first reaches a height of 25 cm?

3 What horizontal distance does the dolphin cover when it next reaches a height of 25 cm? Explain your answer.

4 What horizontal distance does the dolphin cover in one leap? (*Hint:* What is the value of h when the dolphin has completed its leap?)

5 Can this dolphin reach a height of:

a 0.5 m? b 1 m during a leap?

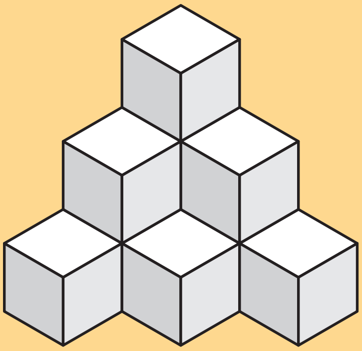
How can you work this out without actually solving the equation?

6 Find the greatest height that the dolphin reaches during a leap.

MATHS QUEST CHALLENGE

1 The diagram shows 3 layers in a pattern of cubes.

If this pattern continues, how many cubes will it take to make 10 layers?



2 Dhiba wrote a list of consecutive whole numbers starting with the number 1. She wrote 288 digits. What was the last number she wrote?

3 Find all positive integers, a, which make the expression $(a - 10)(a + 14)$ a perfect square. Consider 0 to be the first perfect square.

4 Repeat question 3 for the expression $(a - 6)(a + 14)$.

5 If x is the sum of two perfect squares so that $x = a^2 + b^2$, where a and b are whole numbers, explain why $2x$ must also be the sum of two perfect squares.

summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

1 When two or more brackets are multiplied, the process of removing the brackets is called ___________.

2 When expanding two brackets use ___________.

3 Use the rule $(a + b)^2 = a^2 + 2ab + b^2$ to expand ___________ squares.

4 ___________ is the reverse of expansion.

5 Always look for a ___________ when you begin to factorise.

6 An expression where the highest index is 2 is a ___________ expression.

7 An expression with two terms may be able to be factorised by using the ___________ of two squares rule.

8 To factorise a quadratic trinomial of the form $ax^2 + bx + c$, where $a = 1$, identify the factor pair of c that adds to the ___________ of the x-term. Use this factor pair in two sets of brackets. If $a \neq 1$, identify the ___________ of ac that will add to b. The x-term is then split into two terms and the new expression is factorised by ___________.

9 If a quadratic trinomial cannot be factorised easily, then the completing the ___________ method can be used.

10 A quadratic equation in the form $ax^2 + bx + c = 0$ can be solved by:

a factorising and using the ___________ Law

b using the ___________, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

11 The ___________ indicates the number and type of solutions of a quadratic equation.

12 The number of solutions of a quadratic equation correspond to the number of ___________ obtained when the equation is graphed.

WORD LIST

square	difference	coefficient	x-intercepts
Factorisation	factor pair	common factor	quadratic
discriminant	FOIL	perfect	quadratic formula
expansion	Null Factor	grouping	

CHAPTER review

4A

1 Expand each of the following and simplify where necessary.

a $3x(x - 4)$
b $-7x(3x + 1)$
c $(x - 7)(x + 1)$
d $(2x - 5)(x - 3)$
e $(4x - 1)(3x - 5)$
f $3(x - 4)(2x + 7)$
g $(2x - 5)(x + 3)(x + 7)$
h $(x + 5)(x + 7) + (2x - 5)(x - 6)$
i $(x + 3)(5x - 1) - 2x$

4A

2 Expand and simplify each of the following.

a $(x - 7)^2$
b $(2 - x)^2$
c $(3x + 1)^2$
d $-2(3x - 2)^2$
e $-7(2x + 5)^2$
f $-10(4x - 5)^2$
g $(x + 9)(x - 9)$
h $(3x - 1)(3x + 1)$
i $(5 + 2x)(5 - 2x)$

4B

3 Factorise each of the following.

a $2x^2 - 8x$
b $-4x^2 + 12x$
c $3ax - 2ax^2$
d $(x + 1)^2 + (x + 1)$
e $3(2x - 5) - (2x - 5)^2$
f $(x - 4)(x + 2) - (x - 4)$

4B

4 Factorise each of the following.

a $x^2 - 16$
b $x^2 - 25$
c $2x^2 - 72$
d $3x^2 - 27y^2$
e $4ax^2 - 16ay^2$
f $(x - 4)^2 - 9$

4B

5 Factorise each of the following by grouping.

a $ax - ay + bx - by$
b $7x + ay + ax + 7y$
c $xy + 2y + 5x + 10$
d $mn - q - 2q^2 + 2mnq$
e $pq - 5r^2 - r + 5pqr$
f $uv - u + 9v - 9$
g $a^2 - b^2 + 5a - 5b$
h $d^2 - 4c^2 - 3d + 6c$
i $2 + 2m + 1 - m^2$

4B

6 Factorise each of the following by grouping.

a $4x^2 + 12x + 9 - y^2$
b $49a^2 - 28a + 4 - 4b^2$
c $64s^2 - 16s + 1 - 3t$

4C

7 Factorise each of the following.

a $x^2 + 10x + 9$
b $x^2 - 11x + 18$
c $x^2 - 4x - 21$
d $x^2 + 3x - 28$
e $-x^2 + 6x - 9$
f $3x^2 + 33x - 78$
g $-2x^2 + 8x + 10$
h $-3x^2 + 24x - 36$
i $8x^2 + 2x - 1$
j $6x^2 + x - 1$
k $8x^2 + 4x - 12$
l $105x^2 - 10x - 15$
m $-12x^2 + 62x - 70$
n $-45x^2 - 3x + 6$
o $-60x^2 - 270x - 270$

4C

8 The area of the pool at right is $(6x^2 + 11x + 4)$ m^2. Find the length of the rectangular pool if its width is $(2x + 1)$ m.

4D

9 Factorise each of the following by completing the square.
a $x^2 + 6x + 1$
b $x^2 - 10x - 3$
c $x^2 + 4x - 2$
d $x^2 - 5x + 2$
e $x^2 + 7x - 1$
f $2x^2 + 18x - 2$

4E

10 Factorise each of the following using the most appropriate method.
a $3x^2 - 12x$
b $x^2 + 6x + 2$
c $4x^2 - 25$
d $2x^2 + 9x + 10$
e $2ax + 4x + 3a + 6$
f $-3x^2 - 3x + 18$

4E

11 First factorise then simplify each of the following.
a $\dfrac{x+4}{5x-30} \times \dfrac{2x-12}{x+1}$
b $\dfrac{3x+6}{4x-24} \times \dfrac{7x-42}{6x+12}$
c $\dfrac{x^2-4}{x^2+5x} \times \dfrac{x^2+4x-5}{x^2-2x-8}$

4F

12 Solve each of the following quadratic equations by first factorising the left-hand side of the equation.
a $x^2 + 8x + 15 = 0$
b $x^2 + 7x + 6 = 0$
c $x^2 + 11x + 24 = 0$
d $x^2 + 4x - 12 = 0$
e $x^2 - 3x - 10 = 0$
f $x^2 + 3x - 28 = 0$
g $x^2 - 4x + 3 = 0$
h $x^2 - 11x + 30 = 0$
i $x^2 - 2x - 35 = 0$

4F

13 Solve each of the following quadratic equations.
a $2x^2 + 16x + 24 = 0$
b $3x^2 + 9x + 6 = 0$
c $4x^2 + 10x - 6 = 0$
d $5x^2 + 25x - 70 = 0$
e $2x^2 - 7x - 4 = 0$
f $6x^2 - 8x - 8 = 0$
g $2x^2 - 6x + 4 = 0$
h $6x^2 - 25x + 25 = 0$
i $2x^2 + 13x - 7 = 0$

4F

14 Solve each of the following by completing the square. Give an exact answer for each one.
a $x^2 + 8x - 1 = 0$
b $3x^2 + 6x - 15 = 0$
c $-4x^2 - 3x + 1 = 0$

4F

15 Ten times an integer is added to seven times its square. If the result is 152, find the number.

4G

16 Solve each of the following by using the quadratic formula, rounding answers to 3 decimal places.
a $4x^2 - 2x - 3 = 0$
b $7x^2 + 4x - 1 = 0$
c $-8x^2 + x + 2 = 0$

4G

17 Solve each of the following equations, rounding answers to 3 decimal places.

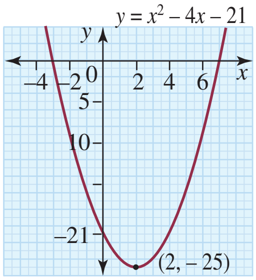
a $18x^2 - 2x - 7 = 0$

b $29x^2 - 105x - 24 = 0$

c $-5x^2 + 2 = 0$

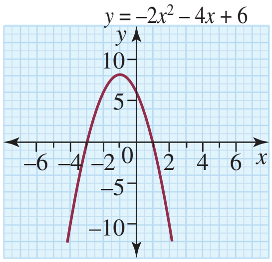
4H

18 The graph of $y = x^2 - 4x - 21$ is shown. Use the graph to find the solutions to the quadratic equation $x^2 - 4x - 21 = 0$.



4H

19 Determine the roots of the quadratic graph shown.



4I

20 Identify whether each of the equations below has no real solutions, one solution or two solutions. State whether the solutions are rational or irrational.

a $x^2 + 11x + 9 = 0$

b $3x^2 + 2x - 5 = 0$

c $x^2 - 3x + 4 = 0$

test yourself CHAPTER 4